Light-cone gauge superstring field theory in linear dilaton background

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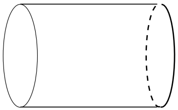
Light-cone gauge closed super SFT

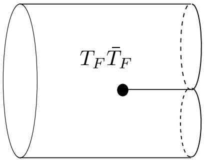
$$S = \int \left[\frac{1}{2} \Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - 1}{p^+} \right) \Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$

Mandelstam, Sin, Green-Scharz-Brink, Gross-Periwal, ...

- No gauge invariance, no Lorentz invariance
- Only three string interaction terms

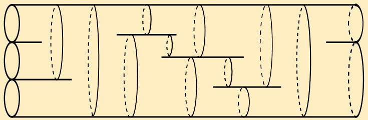


propagator



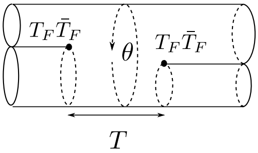
vertex

Feynman amplitudes diverge



- Contact term (CT) divergences
  - Even the tree amplitudes are ill-defined

$\int dT d\theta$



$T \to 0$

$\theta \to \theta_0$

- Degenerations of the worldsheet also cause divergences.

We would like to get finite amplitudes

Strategy

We regularize the amplitudes, by considering the SFT in linear dilaton background

$$\Phi = -iQX^1$$

We would like to show

1. The amplitudes become finite for $Q^2 > 10$.
2. The amplitudes coincide with those obtained by the 1-st quantized approach in the limit $Q \to 0$.

under certain assumptions.

Based on Murakami-N.I. arXiv:1063.008337,

N.I. arXiv:106504666, 1066*******

Outline

1. LC gauge super SFT in LD background
2. Divergences
3. Finiteness
4. Comparison with the first quantized approach
5. Conclusions and discussions

§1 LC gauge super SFT in LD background

Linear dilaton background $\Phi = -iQX^1$ $(ds^2 = 2\hat{g}_{z\bar{z}}dzd\bar{z})$

$$S = \frac{1}{16\pi}\int dz \wedge d\bar{z} i\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a X^1 \partial_b X^1 - 2iQ\hat{R}X^1\right)$$

- $i\partial\tilde{X}^1(z) \equiv i\partial(X^1 - iQ\ln(2g_{z\bar{z}})) = \sum_n \alpha_n^1 z^{-n-1}$
- $\left(\alpha_n^1\right)^* = (-1)^{n+1}(\alpha_{-n}^1 + 2Q\delta_{n,0})$
- $L_0 = \frac{1}{2}p^2 + Qp + N$
- $c = 1 - 12Q^2$
- the correlation functions on the sphere

$$\int \left[dX^1\right]_{g_{z\bar{z}}} e^{-S} \prod_{r=1}^{N} e^{ip_r\tilde{X}^1}(Z_r, \bar{Z}_r) = 2\pi\delta\left(\sum p_r + 2Q\right) e^{-\frac{1-12Q^2}{24}\Gamma} \prod_{r>s} |Z_r - Z_s|^{2p_r p_s}$$

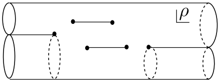
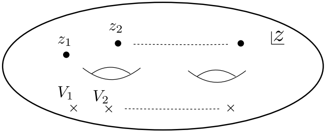
LC gauge super SFT in LD background GO

We construct SFT (type II) with the worldsheet theory for $X^i, \psi^i, \bar{\psi}^i\ (i = 1, \cdots 8)$

$$S = \int \left[\frac{1}{2} \Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - 1 + Q^2 - i\varepsilon}{p^+} \right) \Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$

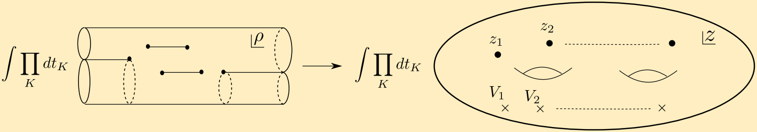
- Feynman amplitude

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}} e^{-(1-Q^2)\Gamma}$$

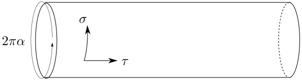
$\int \prod_K dt_K$  $\longrightarrow \int \prod_K dt_K$ 

Anomaly factor

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| (\partial^2 \rho)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}} e^{-(1-Q^2)\Gamma}$$



- On the LC diagram there is a naturally defined metric $ds^2 = d\rho d\bar{\rho}$

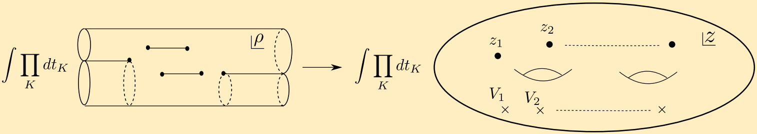


$\rho = \tau + i\sigma$

- $ds^2 = d\rho d\bar{\rho}$ is singular at $z_1, \cdots$ and punctures.

Anomaly factor

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2\rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}\left(z_I\right) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}} e^{-(1-Q^2)\Gamma}$$

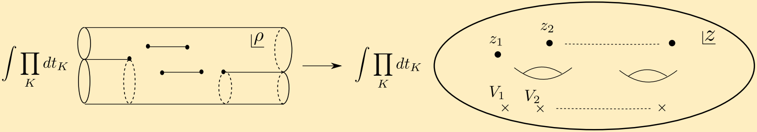


- The integrand of A_Q^{LC} is defined with the metric $ds^2 = d\rho d\bar{\rho}$.

$$\begin{aligned} A_Q^{\mathrm{LC}} &= \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2\rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}\left(z_I\right) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{\partial\rho\bar{\partial}\bar{\rho}} \\ &= \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2\rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}\left(z_I\right) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}} e^{-(1-Q^2)\Gamma} \end{aligned}$$

Anomaly factor

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}\left(z_I\right) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}} e^{-(1-Q^2)\Gamma}$$



- $e^{-\Gamma}$ was calculated by Mandelstam (tree bosonic), Berkovits (tree super), Murakami-N.I. (multiloop) GO

For large Q^2, $e^{-(1-Q^2)\Gamma}$ has the effect of taming divergences.

§2 Divergences

The amplitude is expressed explicitly in terms of the theta functions defined on the Riemann surface.

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}^{\mathrm{A}}} e^{-(1-Q^2)\Gamma}$$

$$e^{-\Gamma} \propto \prod_{r=1}^{N} \left[\alpha_r^{-1} (g_{Z_r \bar{Z}_r}^{\mathrm{A}})^{-\frac{1}{2}} e^{-\mathrm{Re}\bar{N}_{00}^{rr}} \right] \prod_{I=1}^{2g-2+N} \left[(g_{z_I \bar{z}_I}^{\mathrm{A}})^{-\frac{1}{2}} \left| \partial^2 \rho(z_I) \right|^{-\frac{1}{2}} \right]$$

$$\rho(z) = \sum_{r=1}^{N} p_r^{+} \left[\ln E(z, Z_r) - 2\pi i \int_{P_0}^{z} \omega \frac{1}{\mathrm{Im}\Omega} \mathrm{Im} \int_{P_0}^{Z_r} \omega \right]$$

$$Z^X [g_{z\bar{z}}^{\mathrm{A}}] = \left| \left(\frac{\prod_i E(z_i, R)\, \sigma(R) \det \omega_\nu z_i}{\vartheta(e_\nu) \prod_{i>j} E\left(z_i, z_j\right) \prod_i \sigma(z_i)} \right)^{\frac{1}{3}} \right|^2 e^{-S}$$

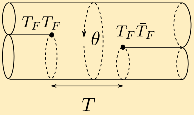
$$\left\langle \prod_i e^{i q_i H}(z_i) \right\rangle = \vartheta \begin{bmatrix} \alpha' \\ \alpha'' \end{bmatrix} (e_\nu) \prod_{i>j} E\left(z_i, z_j\right)^{q_i q_j}$$

From the explicit form of the amplitudes,

we can see that the divergences arise when

1. Some of the interaction points collide with each other. (CT divergences)

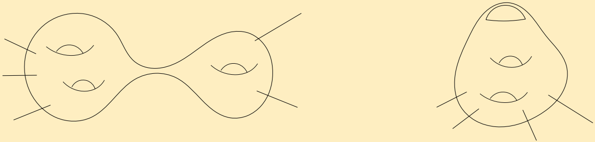
$\int dT d\theta$



$T \to 0$

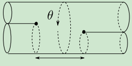
$\theta \to \theta_0$

2. The Riemann surface corresponding to the world sheet degenerates.



Possible divergences arise from the combinations of

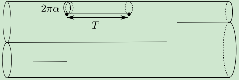
Contact term



$T \to 0$

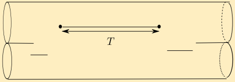
$\theta \to \theta_0$

Infinitely thin cylinder



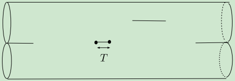
$\alpha \to 0$

Infinitely long cylinder

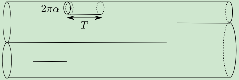


$T \to \infty$

Tiny neck



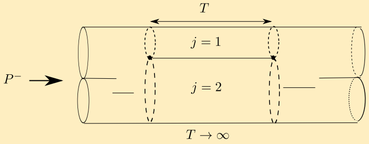
$T \to 0$



$T \sim \alpha \sim \epsilon \sim 0$

§3 Finiteness

Infinitely long cylinder



$$\int_0^\infty dT \exp\left[-T\left(\sum_j \frac{L_0^{(j)} + \bar{L}_0^{(j)} - 1 + Q^2 - i\varepsilon}{\alpha_j} - P^-\right)\right]$$

- Following Berera, Witten, we modify the contour as

$$\int_0^\infty dT \to \left(\int_0^{T_0} + \int_{T_0}^{T_0+i\infty}\right) dT$$

  we get a finite result.

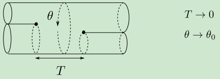
- The Feynman $i\varepsilon$ takes care of the divergences of this kind.
- We assume that taking the limit $\varepsilon \to 0$ causes no problems.

Other kinds of divergences

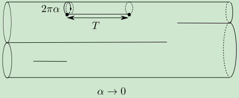
$$A_Q^{\rm LC} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\rm LC}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\rm LC} \right\rangle_{g_{z\bar{z}}^{\rm A}} e^{-(1-Q^2)\Gamma} = \int d^n t F(\vec{t})$$

Examining the behavior of $F(\vec{t})$, we show that the other divergences are tamed by taking Q^2 large enough.

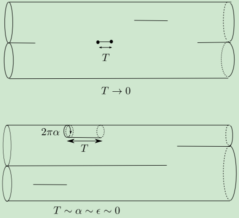
Contact term



Infinitely thin cylinder

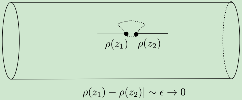


Tiny neck



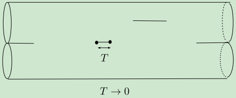
Power counting

Contact term



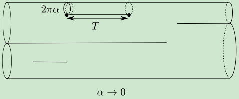
$$F(\vec{t}) \sim \epsilon^{-\frac{10}{3}+\frac{1}{3}Q^2}$$

Tiny neck 1



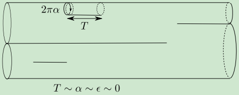
$$F(\vec{t}) \sim T^{-6+Q^2}$$

Infinitely thin cylinder



$$F(\vec{t}) \sim \alpha^{-6+Q^2+\frac{D}{2}}$$

Tiny neck 2



$$F(\vec{t}) \sim \epsilon^{-6+3Q^2}$$

Finiteness

- For $Q = 0$, $F(\vec{t})$ becomes singular at the points corresponding to these configurations
- For Q^2 large enough, $F(\vec{t})$ becomes regular at these points.

For $Q^2 > 10$, we find

- $F(\vec{t})$ is a continuous function without singularities.
- $A_Q^{\mathrm{LC}} = \int d^n t F(\vec{t})$ is finite.

§4 Comparison with the first quantized approach

- The divergences of the amplitudes are regularized by taking $\Phi = -iQX^1$, with $Q^2 > 10$.
- We can define the amplitudes for $Q^2 > 10$ as analytic functions of Q

$$A_Q^{\mathrm{LC}} = \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| (\partial^2 \rho)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g_{z\bar{z}}^{\mathrm{A}}} e^{-(1-Q^2)\Gamma}$$

and take the the limit $Q \to 0$.

We would like to compare the results with those of the first quantized approach.

Conformal gauge expression

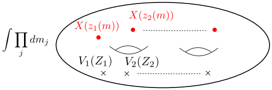
The LC amplitude can be recast into a conformal gauge expression (even spin structure)

$$
\begin{aligned}
A_Q^{\mathrm{LC}} &= \int \prod_K dt_K \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2\rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{g^{\mathrm{A}}_{z\bar{z}}} e^{-(1-Q^2)\Gamma} \\
&= \int \prod_j dm_j \left\langle \prod_j \oint (\mu_j b + \bar{\mu}_j \bar{b}) \prod_{I=1}^{2g-2+N} X(z_I)\bar{X}(\bar{z}_I) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle^{X^\mu, \psi^\mu, \mathrm{ghosts}}
\end{aligned}
$$

- with a nontrivial CFT for $X^\pm, \psi^\pm$ ($X^\pm$ CFT). (Murakami-N.I.) GO
- $X(z) = -e^{\phi}G + c\partial\xi + \frac{1}{4}\partial b\eta e^{2\phi} + \frac{1}{4}\left(2\partial\eta e^{2\phi} + \eta\partial e^{2\phi}\right)$: picture changing operator (PCO)
- PCO's are placed at the interaction points.

First quantized approach (Verlinde-Verlinde)

$$A_Q^{VV} = \int_{\mathcal{M}} \prod_j dm_j \left\langle \prod_j \oint (\mu_j b + \bar{\mu}_j \bar{b}) \prod_i^{2g-2+N} X(z_i(m)) \bar{X}(\bar{z}_i(m)) \prod_{r=1}^{N} V_r^{\text{conf.}} \right\rangle$$



- If the PCO's are placed at $z = z_i(m)$, but the amplitudes suffer from the so called spurious singularities.

- Sen-Witten gave a prescription to write down amplitudes placing PCO's avoiding the spurious singularities patchwise.

$$A_Q^{SW} = \sum_\alpha \int_{\mathcal{M}^\alpha} \prod_j dm_j \left\langle \prod_j \oint (\mu_j b + \bar{\mu}_j \bar{b}) \prod_i^{2g-2+N} X(z_i(m)) \bar{X}(\bar{z}_i(m)) \prod_{r=1}^{N} V_r^{\text{conf.}} \right\rangle + \cdots$$

$A_Q^{\rm LC} = A_Q^{SW}$

- When $Q^2 > 10$,

$$
\begin{aligned}
A_Q^{SW} &= \sum_\alpha \int_{\mathcal{M}^\alpha} \prod_j dm_j \left\langle \prod_j \oint \left(\mu_j b + \bar{\mu}_j \bar{b}\right) \prod_i^{2g-2+N} X(z_i(m)) \bar{X}(\bar{z}_i(m)) \prod_{r=1}^N V_r^{\rm conf.} \right\rangle \\
&\quad + \cdots \\
&= \int_{\mathcal{M}} \prod_j dm_j \left\langle \prod_j \oint \left(\mu_j b + \bar{\mu}_j \bar{b}\right) \prod_{I=1}^{2g-2+N} X(z_I) \bar{X}(\bar{z}_I) \prod_{r=1}^N V_r^{\rm conf.} \right\rangle \\
&= A_Q^{\rm LC}
\end{aligned}
$$

because

- putting $z_i(m) = z_I$ does not make the amplitude diverge
- Sen-Witten prescription does no depend on the choice of $z_i(m)$

Therefore as an analytic function of Q, $A_Q^{\rm LC} = A_Q^{SW}$.

We can get $\lim_{Q\to 0} A_Q^{\rm LC} = A_0^{SW}$, if A_0^{SW} is well-defined.

§5 Conclusions and discussions

- In order to regularize the Feynman amplitudes, we consider light-cone gauge superstring field theory in linear dilaton background $\Phi = -iQX^1$.
- The amplitudes become finite for $Q^2 > 10$ and they can be defined as analytic functions of Q. The amplitudes without the background is given by the limit $Q \to 0$.
- The results coincide with those from the first quantized approach.

Outlook

- Equivalence of the amplitudes with odd spin structure.
- Our approach looks quite similar to the dimensional regularization in field theory, but there are crucial differences:

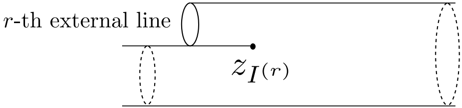
- The number of $\psi^i, \bar{\psi}^i$ is not changed. Therefore the number of the gamma matrices is not changed and we do not have any problems with fermions.
- We have a concrete theory for $Q \neq 0$. It may be possible to discuss nonperturbative problems using this approach.

Anomaly factor

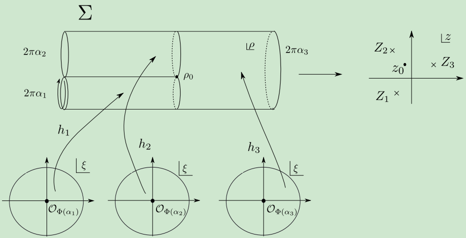
▸ BACK

$$e^{-\Gamma} \propto \prod_{r=1}^{N} \left[\alpha_r^{-1} (g^{\mathrm{A}}_{Z_r \bar{Z}_r})^{-\frac{1}{2}} e^{-\mathrm{Re}\bar{N}^{rr}_{00}} \right] \prod_{I=1}^{2g-2+N} \left[(g^{\mathrm{A}}_{z_I \bar{z}_I})^{-\frac{1}{2}} \left| \partial^2 \rho(z_I) \right|^{-\frac{1}{2}} \right]$$

- $r = 1, \ldots, N$ label the punctures
- $I = 1, \ldots, 2g-2+N$ label the interaction points, where $\partial\rho(z_I) = 0$.
- $g^{\mathrm{A}}_{z\bar{z}}$: Arakelov metric on the surface
- $\bar{N}^{rr}_{00} \equiv \frac{1}{p_r^+} \left(\rho(z_{I(r)}) - \lim_{z \to Z_r} \left(\rho(z) - p_r^+ \ln(z - Z_r) \right) \right)$



Three-string vertex ▸ BACK

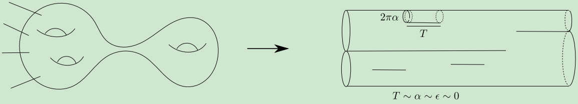


$$\int \Phi_1 \cdot (\Phi_2 * \Phi_3) = \int dt \prod_{r=1}^{3} \left(\frac{p_r^+ dp_r^+}{4\pi} \right) \delta \left(\sum_{r=1}^{3} p_r^+ \right)$$
$$\times \left(p_1^+ p_2^+ p_3^+ \right)^{-\frac{1}{2}(1-Q^2)} e^{-(1-Q^2) \sum_r \frac{1}{p_r^+} \sum_{s=1}^3 p_s^+ \ln\left|p_s^+\right|}$$
$$\times \left\langle \left| \partial^2 \rho \left(z_0 \right) \right|^{-\frac{3}{2}} T_F^{\mathrm{LC}} \left(z_0 \right) \bar{T}_F^{\mathrm{LC}} \left(\bar{z}_0 \right) \right.$$
$$\left. \times \rho^{-1} h_1 \circ \mathcal{O}_{\Phi_1(t,\alpha_1)} \rho^{-1} h_2 \circ \mathcal{O}_{\Phi_2(t,\alpha_2)} \rho^{-1} h_3 \circ \mathcal{O}_{\Phi_3(t,\alpha_3)} \right\rangle_{\mathbb{C}}$$

Remark ▸ BACK

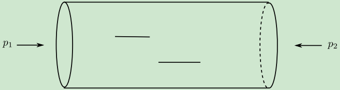
Tadpoles and mass renormalization are irrelevant to the limit $\varepsilon \to 0$.

- Tadpoles: belong to the "Tiny neck" category



- Mass renormalization: If p_1 is on-shell, p_2 is generically off-shell for $Q \neq 0$.

$$p_1^1 + p_2^1 + 2Q(1-g) = 0$$



$X^{\pm}$ CFT

$$\begin{aligned} S_{X^{\pm}} &= -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}X^{+}DX^{-}+\bar{D}X^{-}DX^{+}\right)-Q^2\Gamma_{\text{super}}\left[\Phi\right] \\ & X^{\pm}\equiv x^{\pm}+i\theta\psi^{\pm}+i\bar{\theta}\tilde{\psi}^{\pm}+i\theta\bar{\theta}F^{\pm} \\ & \Gamma_{\text{super}}\left[\Phi\right]=-\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}\Phi D\Phi+\theta\bar{\theta}\hat{g}_{z\bar{z}}\hat{R}\Phi\right) \\ & \Phi\equiv\ln\left|\partial X^{+}-\frac{\partial DX^{+}DX^{+}}{(\partial X^{+})^2}\right|^2-\ln\hat{g}_{z\bar{z}} \end{aligned}$$

- This theory can be formulated in the case $\langle\partial_m X^{+}\rangle\neq 0$.
- In the case of the LC gauge amplitudes, we always have $\prod e^{-ip_r^{+}X^{-}}$ $(p_r^{+}\neq 0)$ and $\langle\partial_m X^{+}\rangle\neq 0$.

$X^{\pm}$ CFT ▸ BACK

$$
\begin{aligned}
S_{X^{\pm}} &= -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}X^{+}DX^{-}+\bar{D}X^{-}DX^{+}\right)-Q^2\Gamma_{\text{super}}\left[\Phi\right] \\
T\left(z,\theta\right) &= G\left(z\right)+\theta T\left(z\right) \\
&= \frac{1}{2}:\partial X^{+}DX^{-}\left(\mathbf{z}\right):+\frac{1}{2}:DX^{+}\partial X^{-}\left(\mathbf{z}\right):+2Q^2S\left(\mathbf{z},\boldsymbol{X}^{+}\right)
\end{aligned}
$$

- It is a superconformal field theory with $\hat{c}=2+8Q^2$.
- The worldsheet theory becomes BRST invariant

$$
\begin{array}{ccccccccc}
 & & X^{\pm} & & X^{i} & & \text{ghosts} & & \\
\hat{c} & = & 2+8Q^2 & + & 8-8Q^2 & - & 10 & = & 0
\end{array}
$$