Light-cone gauge string field theory and dimensional regularization

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Dimensional regularization in string theory?

- Why regularization?
  Superstring theory is UV finite. Why do we need regularization?

- Dimensional regularization?
  The theory should be formulated in the critical dimensions.
  Dimensional regularization should be impossible.

Dimensional regularization in string theory?

- Why regularization?
  Superstring theory is UV finite. Why do we need regularization?
  We use the dimensional regularization to deal with so-called "contact term problem".

- Dimensional regularization?
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Dimensional regularization in string theory?

- Why regularization?
  Superstring theory is UV finite. Why do we need regularization?
  We use the dimensional regularization to deal with so-called "contact term problem".

- Dimensional regularization?
  The theory should be formulated in the critical dimensions.
  Dimensional regularization should be impossible.
  We consider dimensional regularization of LC gauge SFT. It provides a Lorentz noninvariant but gauge invariant regularization.

In this talk

I would like to explain

1. What were the problems/questions? ($\sim \frac{3}{4}$ of the talk)
   - SuperSFT perturbation theores suffer from the contact term problem.
   - This problem is related to the problems of superstring perturbation theory much discussed in 1980's.
   - Recently, Witten gave a way to define the amplitudes without any ambiguities.
2. What is the answer we propose? ($\sim \frac{1}{4}$ of the talk)
   - In the case of LC SFT, the contact term problem can be dealt with by using the dimensional regularization.

Based on collaborations with Baba and Murakami.

Outline

1. Contact term problem

2. Problems about superstring perturbation theory

3. Supermoduli space

4. Dimensional regularization of light-cone gauge SFT

5. Outlook

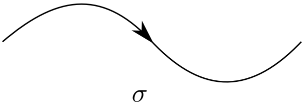
§1 Contact term problem

String perturbation theory from SFT

Example: Witten's cubic SFT (bosonic) (1986)

$$S = \int \left[\frac{1}{2} \Psi Q \Psi + \frac{g}{3} \Psi \cdot (\Psi * \Psi) \right]$$

- String field: $\Psi \left[X^{\mu}(\sigma), b(\sigma), c(\sigma) \right]$



$0 \leq \sigma \leq \pi$

Perturbation theory of bosonic strings

Taking the Siegel gauge $b_0\Psi = 0$,

- gauge fixed action

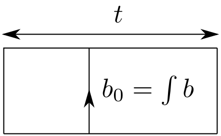
$$S = \int \left[\frac{1}{2} \Psi' c_0 L_0 \Psi' + \frac{g}{3} \Psi' \cdot (\Psi' * \Psi') \right]$$

- Feynman rule

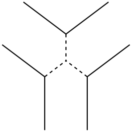
propagator

$\frac{b_0}{L_0} = b_0 \int_0^\infty dt e^{-tL_0}$

$\int_0^\infty dt$

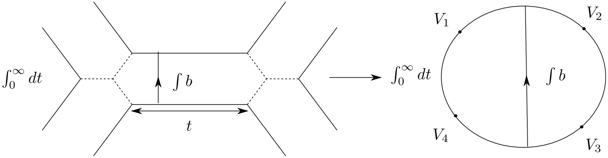


vertex



Feynman diagram

Four point tree amplitude



General amplitudes are expressed in the form

$$A_N = \sum_{\text{worldsheet}} \int \prod_\alpha dt_\alpha \left\langle V_1 \cdots V_N \prod_\alpha \int_{C_\alpha} b \right\rangle_{\text{worldsheet}}$$

with t_α: Feynman parameters

Amplitudes from the first-quantized formalism

GO

$$A = \sum_{\text{worldsheet}} \int \frac{[dg_{mn} dX^\mu]}{\text{rep.} \times \text{Weyl}} e^{-I} V_1 \cdots V_N$$
$$= \sum_{\text{worldsheet}} \int \prod_\alpha dm_\alpha \, [dX^\mu dbdc] \, e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha$$

- $\frac{\text{space of } g_{mn}}{\text{rep.} \times \text{Weyl}}$ =moduli space of worldsheet Riemann surface
- m_α: coordinates of the moduli space GO
- B_α: antighost insertions to soak up the zero modes:

$$B_\alpha = \int d^2\sigma \sqrt{g} \frac{\partial g^{\text{rep.}}_{mn}}{\partial m_\alpha} b^{mn}$$

First-quantized formalism

$$A = \sum_{\text{worldsheet}} \int \prod_\alpha dm_\alpha \, [dX^\mu dbdc] \, e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha$$

- This coincides with the SFT result:

$$A = \sum_{\text{worldsheet}} \int \prod_\alpha dt_\alpha \left\langle V_1 \cdots V_N \prod_\alpha \int_{C_\alpha} b \right\rangle$$

- The Feynman parameters $t_\alpha$of SFT parametrize the moduli space of Riemann surfaces. (.....,Zwiebach 1991)
- $B_\alpha = \int d^2\sigma \sqrt{g} \frac{\partial g^{\text{rep.}}_{mn}}{\partial t_\alpha} b^{mn} = \int_{C_\alpha} b$

Superstring perturbation theory

Witten's cubic SFT for superstrings (1987)

$$S = \int \left[\frac{1}{2} \Psi Q \Psi + \frac{g}{3} \Psi \cdot X \left(\frac{\pi}{2} \right) (\Psi * \Psi) \right] + \text{fermions}$$

- $X(\sigma)$: picture changing operator

$$X(\sigma) = \delta(\beta) G(\sigma) + \cdots$$

$G(\sigma)$: worldsheet supercharge

$$G = \psi^{\mu} i \partial X_{\mu} + \text{ghost part}$$

Superstring perturbation theory

Taking the Siegel gauge $b_0\Psi = 0$

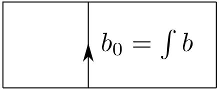
- gauge fixed action

$$S = \int \left[\frac{1}{2}\Psi' c_0 L_0 \Psi' + \frac{g}{3}\Psi' \cdot X\left(\frac{\pi}{2}\right)(\Psi' * \Psi') \right]$$

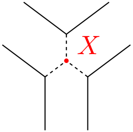
- Feynman rule

propagator

$\frac{b_0}{L_0} = b_0 \int_0^\infty dt e^{-tL_0}$

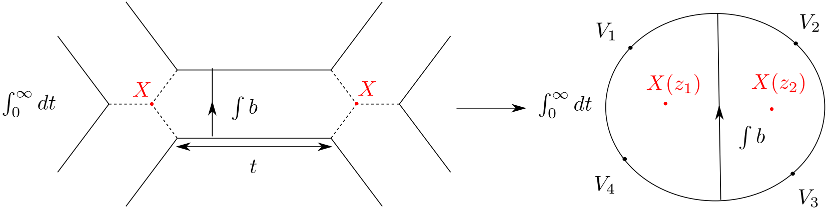
$\int_0^\infty dt$ 

vertex



Superstring perturbation theory

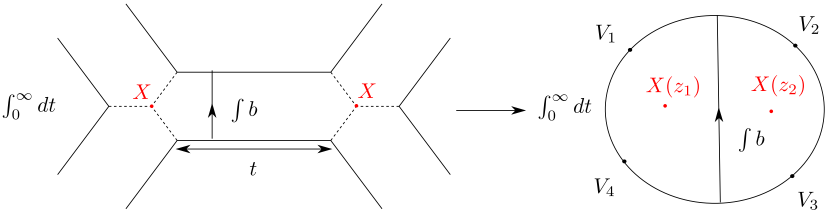
Four point tree amplitude



$$A = \int_0^\infty dt \left\langle V_1 \cdots V_4 X\left(z_1\left(t\right)\right) X\left(z_2\left(t\right)\right) \int b \right\rangle + \text{other channels}$$

Superstring perturbation theory

Four point tree amplitude



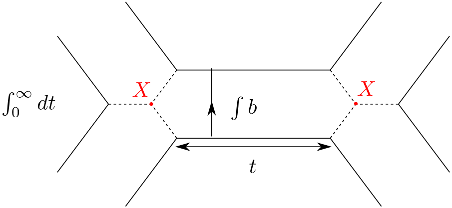
$$A = \int_0^\infty dt \left\langle V_1 \cdots V_4 X\left(z_1\left(t\right)\right) X\left(z_2\left(t\right)\right) \int b \right\rangle + \text{other channels}$$

The integral diverges because $z_1\left(0\right) = z_2\left(0\right)$ and

$$X\left(z_1\right) X\left(z_2\right) \sim \left(z_1 - z_2\right)^{-2} \times \text{regular operator } \left(z_1 \sim z_2\right)$$

Contact term problem

The superSFT yields obviously a wrong answer

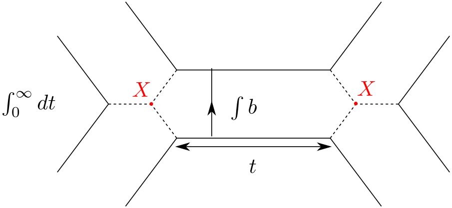


- Even the four point tree amplitude is divergent, because the picture changing operators come close to each other
- This phenomenon is ubiquitous. Amplitudes generically diverge.

This is called the contact term problem. (Wendt, 1987)

§2 Problems about superstring perturbation theory

- The amplitudes from the superstring field theory are obviously wrong. We need to modify the action so that it reproduce the right (the first-quantized) results.



- Actually the first-quantized formalism also has problems in multi-loop calculations.
The contact term problem can be discussed in the context of the problems of first quantized superstring perturbation theory.

Amplitudes from the first-quantized formalism

Martinec, Chaudhuri-Kawai-Tye GO

$$A = \sum_{\text{worldsheet}} \int \frac{[dg_{mn} d\chi_a dX^\mu d\psi^\mu]}{\text{superrep.} \times \text{superWeyl}} e^{-I} V_1 \cdots V_N$$

$$= \sum_{\text{worldsheet}} \int \prod_\alpha dm_\alpha \prod_\sigma d\eta_\sigma \left[dX^\mu d\psi^\mu db dc d\beta d\gamma\right]$$

$$\times e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha \prod \delta\left(\beta_\sigma\right)$$

- $\frac{\text{space of } g_{mn}, \chi_a}{\text{superrep.} \times \text{superWeyl}}$ =supermoduli space of superRiemann surface
- m_α,η_σ: coordinates of the supermoduli space GO
- $B_\alpha, \delta\left(\beta_\sigma\right)$: antighost insertions to soak up the zero modes

$$\beta_\sigma = \int d^2 z \frac{\partial \chi_{\bar{z}}^{\theta \text{rep.}}}{\partial \eta_\sigma} \beta$$

Picture changing operator

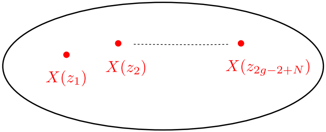
Verlinde-Verlinde

If one takes η_σ so that $\frac{\partial \chi_{\bar{z}}^{\theta \text{rep.}}}{\partial \eta_\sigma} = \delta^2(z - z_\sigma)$ and integrating over η_σ we get

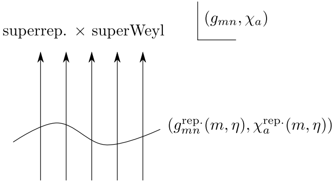
$$
\begin{aligned}
A &= \sum_{\text{worldsheet}} \int \prod_\alpha dm_\alpha \prod_\sigma d\eta_\sigma \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \\
&\qquad \times e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma \delta(\beta_\sigma) \\
&= \sum_{\text{worldsheet}} \int \prod_\alpha dm_\alpha \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \\
&\qquad \times e^{-I} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma X(z_\sigma)
\end{aligned}
$$

Picture changing operators

- Taking $\frac{\partial \chi_{\bar{z}}^{\theta \mathrm{rep.}}}{\partial \eta_\sigma} = \delta^2(z - z_\sigma)$ we get the amplitudes with picture changing operators inserted.



- We can freely take z_σ as long as $\frac{\partial \chi_{\bar{z}}^{\theta \mathrm{rep.}}}{\partial \eta_\sigma}$ $(\sigma = 1, \cdots 2g - 2 + N)$ span the space transverse to the symmetry orbits. It is like a gauge choice.



SFT amplitude

SFT amplitude

$$A_4 = \int_0^\infty dt \left\langle V_1 \cdots V_4 X\left(z_1\left(t\right)\right) X\left(z_2\left(t\right)\right) \int b \right\rangle$$

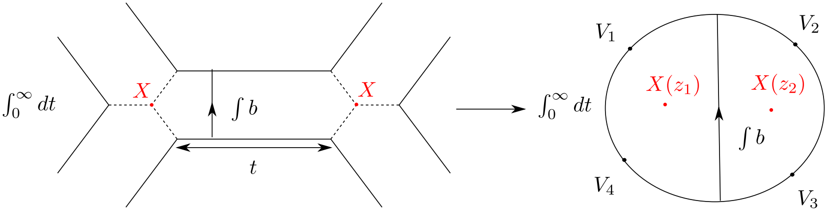
The 1-st quantized result

$$A = \sum_{\text{worldsheet}} \int \prod_\alpha dm_\alpha \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \times e^{-I} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma X\left(z_\sigma\right)$$

The SFT amplitude corresponds to the specific choice

$$\frac{\partial \chi_{\bar{z}}^{\theta \text{rep.}}}{\partial \eta_\sigma} = \delta^2\left(z - z_\sigma\left(t\right)\right) \ \ \left(\sigma = 1, 2\right)$$

Contact term problem



- SFT amplitude corresponds to the choice $\frac{\partial \chi_{\bar{z}}^{\theta\text{rep.}}}{\partial \eta_\sigma} = \delta^2\left(z - z_\sigma\left(t\right)\right)$ $\sigma = 1, 2$.
- The amplitude diverges at $t = 0$, because $\frac{\partial \chi_{\bar{z}}^{\theta\text{rep.}}}{\partial \eta_1}, \frac{\partial \chi_{\bar{z}}^{\theta\text{rep.}}}{\partial \eta_2}$ do not span the two dimensional space transverse to the symmetry orbit. Namely it is a bad "gauge choice" at $t = 0$.

In order to avoid divergence

Since the "gauge" we choose is not good at $t=0$ in

$$A_4 = \int_0^\infty dt F(t)$$

$$F(t) = \left\langle V_1 \cdots V_4 X(z_1(t)) X(z_2(t)) \int b \right\rangle$$

why don't we take a different gauge for $t \sim 0$, namely a different way to place the picture changing operators:

$$A_4 = \int_a^\infty dt F(t) + \int_0^a dt F'(t) \ ?$$

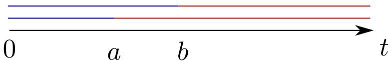
$$F'(t) = \left\langle V_1 \cdots V_4 X(z_1(t) + \Delta z) X(z_2(t)) \int b \right\rangle$$

Total derivative ambiguity

$$A_4(a) = \int_a^\infty dt F(t) + \int_0^a dt F'(t) \ ?$$

This does not work because the expression depends on how we choose a.

$$F'(t) - F(t) = \partial_t f(t)$$



$$A(b) - A(a) = \int_a^b dt \left(F'(t) - F(t)\right) = \int_a^b dt \partial_t f(t) = f(b) - f(a) \neq 0$$

Since there is no canonical way to choose a, the result becomes ambiguous.

Total derivative ambiguity

$$F(t) = \left\langle V_1 \cdots V_4 X(z_1(t)) X(z_2(t)) \int b \right\rangle$$
$$F'(t) = \left\langle V_1 \cdots V_4 X(z_1(t) + \Delta z) X(z_2(t)) \int b \right\rangle$$

Since

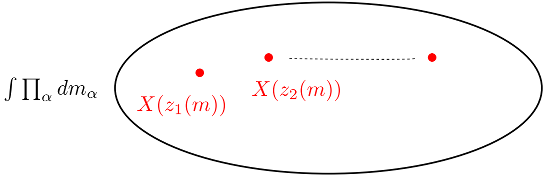
$$X(z_1(t) + c) - X(z_1(t)) = \{Q, \chi(t)\}$$

we get

$$\begin{aligned} F'(t) - F(t) &= \left\langle V_1 \cdots V_4 \{Q, \chi(t)\} X(z_2(t)) \int b \right\rangle \\ &= \partial_t \langle V_1 \cdots V_4 \chi(t) X(z_2(t)) \rangle \\ &\equiv \partial_t f(t) \end{aligned}$$

The problems about superstring perturbation theory

In general we have



- For lower order amplitudes, there is a way to take a good choice of $z_\sigma(m)$ all over the moduli space.
- For higher order amplitudes, this is impossible and the amplitudes become ambiguous.

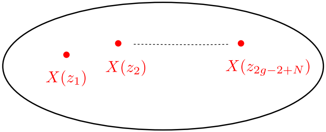
§3 Supermoduli space

$$A_4(a) = \int_a^\infty dtF(t) + \int_0^a dtF'(t) \ ?$$

- The amplitudes become ambiguous because

$$\int_a^b dtF'(t) \neq \int_a^b dtF(t)$$

- The different choice of z_σ corresponds to a different choice of η_σ. $\left(\frac{\partial \chi_{\bar{z}}^{\theta \mathrm{rep.}}}{\partial \eta_\sigma} = \delta^2(z - z_\sigma)\right)$



Supermoduli space

We started from

$$A = \int \prod_{\alpha} dm_{\alpha} \prod_{\sigma} d\eta_{\sigma} \left[dX^{\mu} d\psi^{\mu} db dc d\beta d\gamma\right] \times e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_{\alpha} B_{\alpha} \prod_{\sigma} \delta\left(\beta_{\sigma}\right)$$

$$= \int \prod_{\alpha} dm_{\alpha} \prod_{\sigma} d\eta_{\sigma} \Lambda\left(m, \eta\right)$$

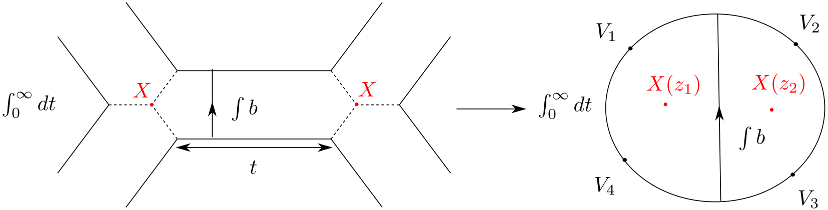
Considered as an integral over the supermoduli space (m, η), it does not depend on the choice of η.

$$\prod_{\alpha} dm'_{\alpha} \prod_{\sigma} d\eta'_{\sigma} = \prod_{\alpha} dm_{\alpha} \prod_{\sigma} d\eta_{\sigma} \text{sdet}\left(\frac{\partial\left(m', \eta'\right)}{\partial\left(m, \eta\right)}\right)$$

$$\Lambda'\left(m', \eta'\right) = \Lambda\left(m, \eta\right)\left(\text{sdet}\left(\frac{\partial\left(m', \eta'\right)}{\partial\left(m, \eta\right)}\right)\right)^{-1}$$

Supermoduli space

Considered as an integral over the supermoduli space, there should not be any ambiguity. (...,D'Hoker-Phong, Witten)



$$A_4 = \int_a^\infty dtF(t) + \int_0^a dtF'(t) + \cdots \ ?$$

Let us rewrite the amplitude as an integral over the supermoduli space and see what happens.

Total derivative ambiguity

$$A_4 = \int_a^\infty dt F(t) + \int_0^a dt F'(t) + \cdots \; ?$$

For $a \leq t$, the integration over the supermoduli space is

$$\begin{aligned} \int dt d\eta_1 d\eta_2 \Lambda(t, \eta_1, \eta_2) &= \int dt d\eta_1 d\eta_2 (H(t) - \eta_1 \eta_2 F(t)) \\ &= \int dt F(t) \end{aligned}$$

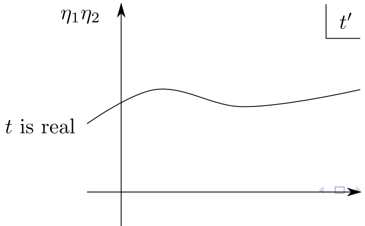
For $0 \leq t \leq a$

$$\begin{aligned} \int dt' d\eta'_1 d\eta'_2 \Lambda'(t', \eta'_1, \eta'_2) &= \int dt' d\eta'_1 d\eta'_2 (H'(t') - \eta'_1 \eta'_2 F'(t')) \\ &= \int dt' F'(t') \end{aligned}$$

Total derivative ambiguity

$$\int dtF(t) = \int dt d\eta_1 d\eta_2 \Lambda(t,\eta_1,\eta_2)$$
$$\int dt' F'(t') = \int dt' d\eta_1' d\eta_2' \Lambda'(t',\eta_1',\eta_2')$$

$$\eta_1' = \eta_1$$
$$\eta_2' = \eta_2$$
$$t' = t + g(t)\eta_1\eta_2$$



Total derivative ambiguity

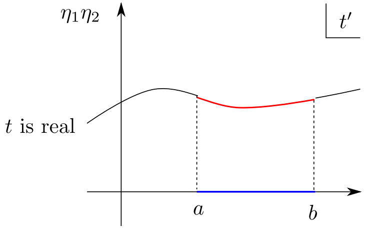
Formally

$$\int dtF(t) = \int dtd\eta_1 d\eta_2 \Lambda(t,\eta_1,\eta_2) = \int dt' d\eta_1' d\eta_2' \Lambda'(t',\eta_1',\eta_2') = \int dt' F'(t')$$

but

$$\int_a^b dtF(t) \neq \int_a^b dt' F'(t')$$

because $a \leq t \leq b$ does not mean $a \leq t' \leq b$.



Total derivative ambiguity

$$\begin{aligned}\int_a^b dt F(t) &= \int_a^b dt d\eta_1 d\eta_2 \Lambda(t, \eta_1, \eta_2) \\ &= \int d\eta_1' d\eta_2' \int_{a+g(a)\eta_1\eta_2}^{b+g(b)\eta_1\eta_2} dt' \Lambda'(t', \eta_1', \eta_2') \\ &= \int d\eta_1' d\eta_2' \int_{a+g(a)\eta_1\eta_2}^{b+g(b)\eta_1\eta_2} dt' \left(H'(t') - \eta_1'\eta_2' F(t')\right) \\ &= \int_a^b dt' F'(t') - gH'(b) + gH'(a)\end{aligned}$$

Therefore

$$\int_a^b dt\left(F'(t) - F(t)\right) = \int_a^b dt \partial_t f(t) = f(b) - f(a)$$

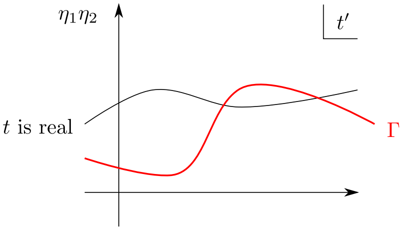
with $f = gH'$ (Atick, Rabin and Sen 1987)

Witten's prescription

The amplitude is given by an integral

$$A = \int_\Gamma dt d\eta_1 d\eta_2 \Lambda\,(t,\eta)$$

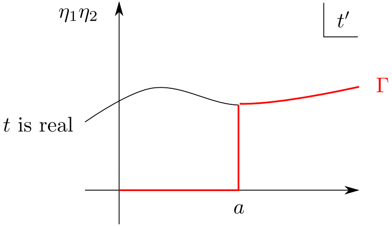
- Γ is a contour of t which can have a nilpotent part.
- Γ is taken to be any contour because Λ is analytic in t, if it behaves well at infinity.



Witten's prescription

$$A = \int_\Gamma dt d\eta_1 d\eta_2 \Lambda(t, \eta)$$

For our purpose, it is convenient to take Γ to be



Witten's prescription

$$A = \int_{\Gamma} dt d\eta_1 d\eta_2 \Lambda(t, \eta)$$

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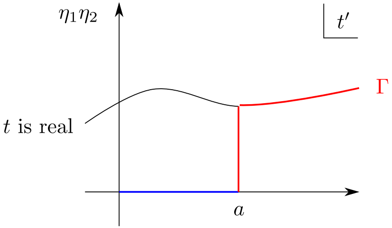
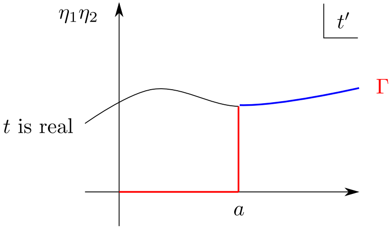


$$\int_0^a dt F'(t)$$

Witten's prescription

$$A = \int_{\Gamma} dt d\eta_1 d\eta_2 \Lambda\left(t, \eta\right)$$

For our purpose, it is convenient to take Γ to be

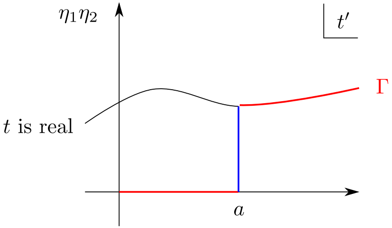


$$\int_a^{\infty} dt F\left(t\right)$$

Witten's prescription

$$A = \int_\Gamma dt d\eta_1 d\eta_2 \Lambda(t,\eta)$$

For our purpose, it is convenient to take Γ to be

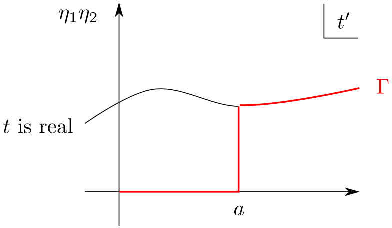


$$\int d\eta_1' d\eta_2' \int_a^{a+g(a)\eta_1\eta_2} dt' \left(H'(t') - \eta_1'\eta_2' F(t')\right) = -gH'(a) = -f(a)$$

Witten's prescription

$$A = \int_{\Gamma} dt d\eta_1 d\eta_2 \Lambda(t, \eta)$$

For our purpose, it is convenient to take Γ to be



$$\int_a^{\infty} dt F(t) + \int_0^a dt F'(t) - f(a)$$

Total derivative ambiguity

So we get the amplitude

$$A_4 = \int_a^\infty dt F(t) + \int_0^a dt F'(t) - f(a)$$

- This does not depend on a

$$\partial_a (A_4) = F'(a) - F(a) - \partial_a f(a) = 0$$

- For $a = \epsilon \ll 1$

$$\begin{aligned} A_4 &= \int_\epsilon^\infty dt F(t) + \int_0^\epsilon dt F'(t) - f(\epsilon) \\ &\sim \int_\epsilon^\infty dt F(t) - f(\epsilon) \end{aligned}$$

$f(\epsilon)$ gives the conterterm to cancel the divergence of $\int_\epsilon^\infty dt F(t)$

General amplitudes

In general, we have the expression

$$\begin{aligned} A &= \int_\Gamma \prod_\alpha dm_\alpha \prod_\sigma d\eta_\sigma \Lambda(m,\eta) \\ &= \sum_U \int_U \prod_\alpha dm_\alpha F(m) + \sum_{\partial U} \int_{\partial U} f_{\partial U} \end{aligned}$$

These expressions are not useful in calculating the amplitudes. It is better if we do not have the second term.

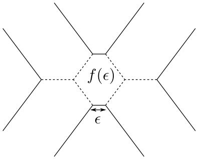
- We can have an expression without the second term if the supermoduli space is projected/split.
- For higher genera, the supermoduli space is not holomorphically projected/split. (Donagi-Witten 2013)

GO

SFT

$$
\begin{aligned}
A_4 &= \int_\epsilon^\infty dt F(t) + \int_0^\epsilon dt F'(t) - f(\epsilon) + \cdots \\
&\sim \int_\epsilon^\infty dt F(t) - f(\epsilon) + \cdots
\end{aligned}
$$

- From the SFT point of view, the counterterm $f(\epsilon)$ corresponds to a 4-string counterterm in the SFT action.



- We need to add 5-string, 6-string... counterterms. This is a disaster for SFT.

The way to deal with the contact term problem

In order to deal with the problem, modifications of Witten's action are proposed:

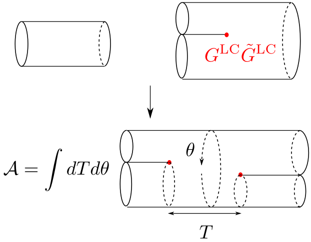
- modified cubic (Preitschopf-Thorn-Yost, Arefeva-Medvedev-Zubarev 1990)
  BRST invariance of multiloop amplitudes
  (Kohriki-Kishimoto-Kugo-Kunitomo-Murata 2011)
- Berkovits (1995)
  BRST invariance of tree amplitudes
  (Kroyter-Okawa-Schnabl-Torii-Zwiebach 2012)

These formulations take the string field to have pictures different from the canonical ones, it will need some work to relate these to the first-quantized results.

§4 Light-cone gauge SFT

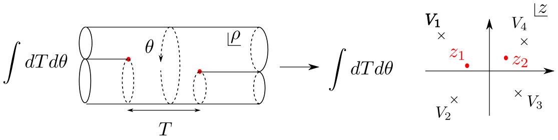
$t = x^+$ (Kaku-Kikkawa, Mandelstam, S.J. Sin)

$$S = \int \left[\frac{1}{2} \Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - \frac{d-2}{8}}{\alpha} \right) \Phi + \frac{g}{6} \Phi \cdot (\Phi * \Phi) \right] \quad (d = 10)$$



The integral diverges.

Light-cone gauge SFT



There exists a procedure to rewrite the LC gauge amplitude into the coformal gauge one. (D'Hoker-Giddings, Kugo-Zwiebach,...)

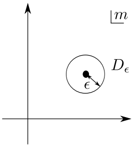
$$
\begin{aligned}
A &= \int dTd\theta \left\langle \prod_{I=1}^{2} \left| \left(\partial^2\rho\right)^{-\frac{3}{4}} G^{\mathrm{LC}}(z_I) \right|^2 \prod_r V_r^{\mathrm{LC}} \right\rangle_{\mathbb{C}}^{X^i} e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&= \int dTd\theta \left\langle \oint \mu b \oint \bar{\mu}\bar{b} \prod_{I=1}^{2} X\bar{X}(z_I, \bar{z}_I) \prod_r V_r^{\mathrm{conf.}} \right\rangle_{\mathbb{C}}^{X^\mu, b, c}
\end{aligned}
$$

This divergence has the same origin as the one in the previous sections.

Light-cone gauge SFT

$$A = \int dT d\theta \left\langle \oint \mu b \oint \bar{\mu}\bar{b} \prod_{I=1}^{2} X\bar{X}\left(z_I, \bar{z}_I\right) \prod_r V_r^{\mathrm{conf.}} \right\rangle_{\mathbb{C}}^{X^\mu, b, c}$$

- The divergence can be dealt with as in the previous section.



$$A = \int_{\mathcal{M}-D_\epsilon} d^2 m F\left(m, \bar{m}\right) + \int_{D_\epsilon} d^2 m F'\left(m, \bar{m}\right) + \int_{\partial D_\epsilon} f$$

Dimensional regularization

Light-cone gauge SFT can be formulated in any d

$$S = \int \left[\frac{1}{2}\Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - \frac{d-2}{8}}{\alpha} \right) \Phi + \frac{g}{6}\Phi \cdot (\Phi * \Phi) \right]$$

- LC gauge SFT is a completely gauge fixed theory.
- The Lorentz invariance is broken.

Dimensional regularization

Even for $d \neq 10$, following the same procedure as that in the previous slide

$$
\begin{aligned}
A &= \int dT d\theta \left\langle \prod_{I=1}^{2} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} G^{\mathrm{LC}}\left(z_I\right) \right|^2 \prod_r V_r^{\mathrm{LC}} \right\rangle_{\mathbb{C}}^{X^i} e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&= \int dT d\theta \left\langle \oint \mu b \oint \bar{\mu}\bar{b} \prod_{I=1}^{2} X\bar{X}\left(z_I, \bar{z}_I\right) \prod_r V_r^{\mathrm{conf.}} \right\rangle_{\mathbb{C}}^{X^\mu, b, c}
\end{aligned}
$$

but with a nontrivial CFT for $X^{\pm}$ ($X^{\pm}$ CFT).

- The worldsheet theory becomes BRST invariant

$$
\begin{array}{ccccccccc}
 & & X^{\pm} & & X^i & & b, c & & \\
\hat{c} & = & 12 - d & + & d - 2 & - & 10 & = & 0
\end{array}
$$

In the second-quantized language, DR is a gauge invariant regularization.

$X^{\pm}$ CFT

$$S_{X^{\pm}} = -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}\mathcal{X}^+ D\mathcal{X}^- + \bar{D}\mathcal{X}^- D\mathcal{X}^+\right) + \frac{d-10}{8}\Gamma_{\text{super}}[\Phi]$$

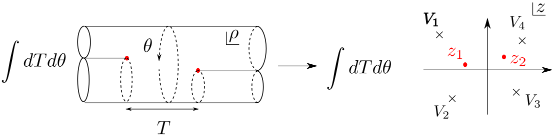
$$\mathcal{X}^{\pm} \equiv X^{\pm} + i\theta\psi^{\pm} + i\bar{\theta}\tilde{\psi}^{\pm} + i\theta\bar{\theta}F^{\pm}$$

$$\Gamma_{\text{super}}[\Phi] = -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\bar{D}\Phi D\Phi$$

$$\Phi \equiv \ln\left(\left(D\Theta^+\right)^2\left(\bar{D}\tilde{\Theta}^+\right)^2\right) \quad \Theta^+ \equiv \frac{D\mathcal{X}^+}{\left(\partial\mathcal{X}^+\right)^{\frac{1}{2}}}$$

- This theory can be formulated for $\langle\partial_m X^+\rangle \neq 0$
- It is a superconformal field theory with $\hat{c} = 12 - d$ so that the total central charge becomes $d - 2 + 12 - d - 10 = 0$.

Dimensional regularization



$$A = \int dT d\theta \left\langle \prod_{I=1}^{2} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} G^{\mathrm{LC}}(z_I) \right|^2 \prod_r V_r^{\mathrm{LC}} \right\rangle_{\mathbb{C}}^{X^i} e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$$

$$= \int dT d\theta \left\langle \oint \mu b \oint \bar{\mu}\bar{b} \prod_{I=1}^{2} X\bar{X}(z_I, \bar{z}_I) \prod_r V_r^{\mathrm{conf.}} \right\rangle_{\mathbb{C}}^{X^\mu, b, c}$$

$$e^{-\frac{d-2}{16}\Gamma} \sim |z_1 - z_2|^{-\frac{d-2}{8}} \text{ for } |z_1 - z_2| \sim 0$$

By taking d to be large and negative, the amplitudes do not diverge.

Dimensional regularization

$$A = \int dT d\theta \left\langle \oint \mu b \oint \bar{\mu}\bar{b} \prod_{I=1}^{2} X\bar{X}\left(z_I, \bar{z}_I\right) \prod_r V_r^{\text{conf.}} \right\rangle_{\mathbb{C}}^{X^\mu, b, c}$$

- For d large and negative, the integral is convergent and coincides with the expression

$$A = \int_{\mathcal{M}-D_\epsilon} d^2 m F\left(m, \bar{m}\right) + \int_{D_\epsilon} d^2 m F'\left(m, \bar{m}\right) + \int_{\partial D_\epsilon} f$$

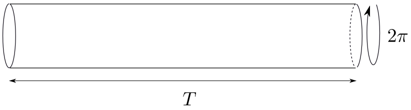
the second and the third term vanishes in the limit $\epsilon \to 0$

- We can define the amplitudes for $d = 10$ by analytic continuation. If the limit $d \to 10$ can be taken without encountering divergences, the results coincides with the usual one.

Remarks

- The dimensional regularization works as a UV/IR regularization

$$S = \int \left[\frac{1}{2} \Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - \frac{d-2}{8}}{\alpha} \right) \Phi + \frac{g}{6} \Phi \cdot (\Phi * \Phi) \right]$$



$$\sim e^{\frac{d-2}{8} T}$$

- What matters is the Virasoro central charge $\hat{c}$ rather than the number of the spacetime coordinates. Therefore we can realize “SFT in fractional dimensions” and the regularization is not restricted to perturbation theory.

Remarks

- We can use super WZW model to deal with Type II theory.
- Dimensional regularization cannot be used to regularize the parity violating amplitudes. We need to break the gauge symmetry to deal with them.
- One can consider similar way of regularization for Witten's superstring field theory.

$$\begin{array}{ccccccccc} & & \varpi, \varphi, \cdots & & X^\mu & & b, c & & \\ \hat{c} & = & 10-d & + & d & - & 10 & = & 0 \end{array}$$

Baba, Murakami, N.I.

§5 Conclusions and discussions

1. We have proposed a way to describe superstring theory by SFT with only three string vertex.
2. With this string field theory it may be possible to describe nonperturbative effects.

Backup

First-quantized formalism

Worldsheet action

$$I = \frac{1}{8\pi} \int d^2\sigma \sqrt{g} g^{mn} \partial_m X^\mu \partial_n X_\mu$$

- reparametrization invariance: $\sigma^m \to \sigma^m + \epsilon^m(\sigma)$
- Weyl invariance: $g_{mn}(\sigma) \to e^{\varepsilon(\sigma)} g_{mn}(\sigma)$

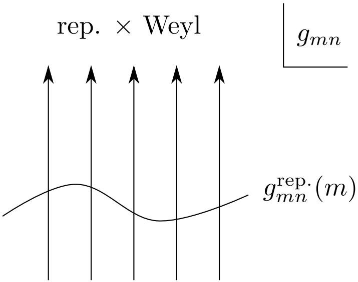
Amplitude

$$A = \sum_{\text{worldsheet}} \int \frac{[dg_{mn} dX^\mu]}{\text{rep.} \times \text{Weyl}} e^{-I} V_1 \cdots V_N$$

- $V_i \ (i = 1, \cdots, N)$: vertex operators

BACK

First-quantized formalism



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First-quantized formalism

Worldsheet action

$$I = \frac{1}{8\pi}\int d^2\sigma\sqrt{g}\,[g^{mn}\partial_m X^\mu \partial_n X_\mu - i\psi^\mu\gamma^m\partial_m\psi_\mu$$
$$-\psi^\mu\gamma^a\gamma^m\chi_a\partial_m X_\mu + \frac{1}{4}\left(\psi^\mu\gamma^a\gamma^b\chi_a\right)\chi_b\psi_\mu\Big]$$

- χ_a: gravitino field on the worlsheet
- superreparametrization invariance and super Weyl invariance

Amplitude

$$A = \sum_{\text{worldsheet}}\int \frac{[dg_{mn}d\chi_a dX^\mu d\psi^\mu]}{\text{superrep.} \times \text{superWeyl}} e^{-I} V_1 \cdots V_N$$

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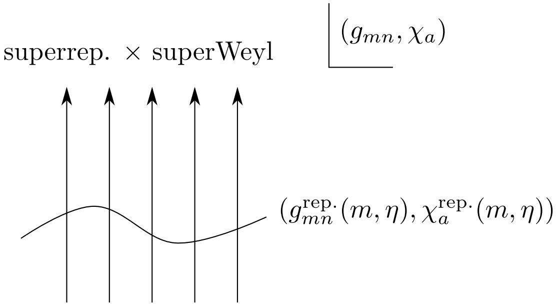
First-quantized formalism



η_σ: odd moduli (Grassmann odd) BACK

Picture changing operator

A convenient choice is $\chi_{\bar{z}}^{(\sigma)\theta} = \delta^2\left(z - z_\sigma\right)$ and $\chi_{\bar{z}}^{\theta} = \sum_\sigma \eta_\sigma \delta^2\left(z - z_\sigma\right)$

$$\begin{aligned} \beta_\sigma &= \int d^2 z \chi_{\bar{z}}^{(\sigma)\theta} \beta_{z\theta} = \beta\left(z_\sigma\right) \\ I_{\text{g.f.}} &= \cdots + \int d^2 z \chi_{\bar{z}}^{\theta} G = I' + \sum_\sigma \eta_\sigma G\left(z_\sigma\right) \end{aligned}$$

BACK

$$\int \prod_\alpha dm_\alpha \prod_\sigma d\eta_\sigma \left[dX^\mu d\psi^\mu db dc d\beta d\gamma\right]$$
$$\times e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma \delta\left(\beta_\sigma\right)$$

Picture changing operator

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BACK

$$\begin{aligned} &\int \prod_\alpha dm_\alpha \prod_\sigma d\eta_\sigma \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \\ &\qquad\qquad \times e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma \delta(\beta_\sigma) \\ &\propto \int \prod_\alpha dm_\alpha \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \\ &\qquad\qquad \times e^{-I'} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma \left(\delta(\beta) G + \cdots\right)(z_\sigma) \end{aligned}$$

Picture changing operator

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$$I_{\text{g.f.}} = \cdots + \int d^2 z \chi_{\bar{z}}^{\theta} G = I' + \sum_\sigma \eta_\sigma G(z_\sigma)$$

BACK

$$\int \prod_\alpha dm_\alpha \prod_\sigma d\eta_\sigma \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \times e^{-I_{\text{g.f.}}} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma \delta(\beta_\sigma)$$

$$\propto \int \prod_\alpha dm_\alpha \left[dX^\mu d\psi^\mu dbdcd\beta d\gamma\right] \times e^{-I'} V_1 \cdots V_N \prod_\alpha B_\alpha \prod_\sigma X(z_\sigma)$$

Projected, split

the supermoduli space is covered by patches with the local coordinates which are related by the transformations of the form

$$\begin{aligned} m'_\alpha &= f_\alpha(m) + \mathcal{O}\left(\eta^2\right) \\ \eta'_\sigma &= \sum_{\sigma'} g_{\sigma\sigma'}(m)\,\eta_{\sigma'} + \mathcal{O}\left(\eta^3\right) \end{aligned}$$

- If one can take the transformations of the form

$$\begin{aligned} m'_\alpha &= f_\alpha(m) \\ \eta'_\sigma &= \sum_{\sigma'} g_{\sigma\sigma'}(m)\,\eta_{\sigma'} + \mathcal{O}\left(\eta^3\right) \end{aligned}$$

  the supermoduli space is projected.

BACK

Projected, split

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\end{aligned}
$$

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$$
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\end{aligned}
$$

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Projected, split

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- If the supermoduli space is projected

$$\begin{aligned} m'_\alpha &= f_\alpha(m) \\ \eta'_\sigma &= \sum_{\sigma'} g_{\sigma\sigma'}(m)\,\eta_{\sigma'} + \mathcal{O}\left(\eta^3\right) \end{aligned}$$

the amplitudes can be expressed

$$\sum_U \int_U \prod_\alpha dm_\alpha F(m)$$

as an integral over the bosonic moduli space.