Light-cone Gauge String Field Theory in Noncritical Dimensions

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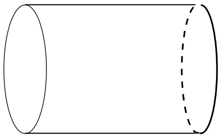
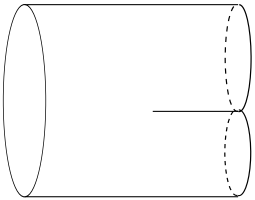
Light-cone gauge SFT (closed)

$$S = \int dt \left[\frac{1}{2} \int d1d2 \langle R(1,2)| \Phi\rangle_1 \left(i\frac{\partial}{\partial t} - H \right) |\Phi\rangle_2 + \frac{2g}{3} \int d1d2d3 \langle V_3(1,2,3)| \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \right]$$





- No gauge symmetry $\longrightarrow$ no need to keep $d = 26$ or 10

CFT for $X^{\pm}$

- No Lorentz symmetry $\longrightarrow$ it should correspond to a string theory in a Lorentz noninvariant background

$$\begin{array}{cccc} & X^i + & \text{ghost}+ & \text{nontrivial CFT for } X^{\pm} \\ c = & d-2 & -26 & 28-d \end{array}$$

- With all these variables we can construct a nilpotent BRST charge.

$$Q_{\mathrm{B}} = \oint \frac{dz}{2\pi i}\left(cT + bc\partial c\right) + \text{c.c.}$$

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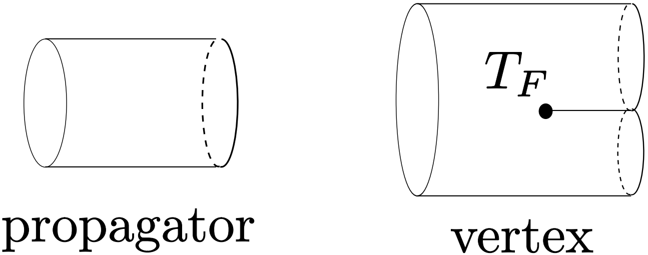
We would like to construct the CFT for the longitudinal variables $X^\pm$. ($X^\pm$ CFT)

Motivation

Light-cone gauge SFT for superstrings (Mandelstam, S.J. Sin)

$$S = \int dt \left[\frac{1}{2} \int d1d2 \left\langle R\left(1,2\right) \right| \Phi\rangle_1 \left(i\frac{\partial}{\partial t} - H \right) |\Phi\rangle_2 + \frac{2g}{3} \int d1d2d3 \left\langle V_3\left(1,2,3\right) \right| \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \right]$$

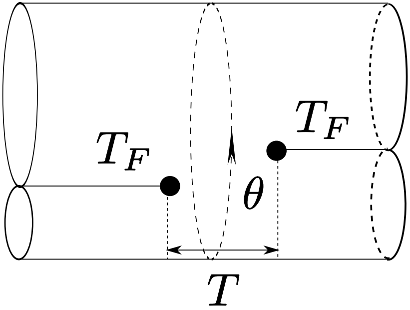


yields divergent results even for tree amplitudes.

Amplitude

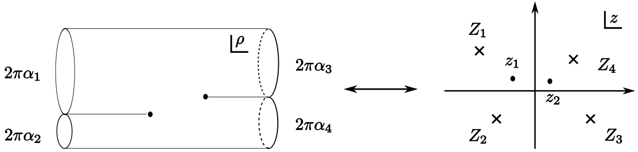
The amplitude diverges when two T_F's come close to each other.

$$\mathcal{A} \sim \int dT d\theta$$



- How can one deal with the divergences in the light-cone gauge SFT?

For general d

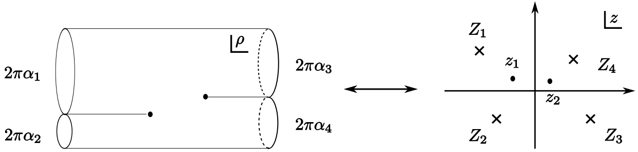


$$ds^2 = d\rho d\bar{\rho} = \partial\rho\bar{\partial}\bar{\rho}dzd\bar{z}$$

$$\mathcal{A} \sim \int d^2\mathcal{T} \left\langle \prod_{I=1,2} \left[T_F^{\mathrm{LC}}(z_I)\,\tilde{T}_F^{\mathrm{LC}}(\bar{z}_I)\right] \prod_{r=1}^{4} V_r^{\mathrm{LC}} \right\rangle$$
$$\times\, e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \prod_{I=1,2} \left(\partial^2\rho\,(z_I)\,\bar{\partial}^2\bar{\rho}\,(\bar{z}_I)\right)^{-\frac{3}{4}}$$

$\Gamma[\phi] = -\frac{1}{\pi}\int d^2z\,\partial\phi\bar{\partial}\phi$: Liouville action

For general d



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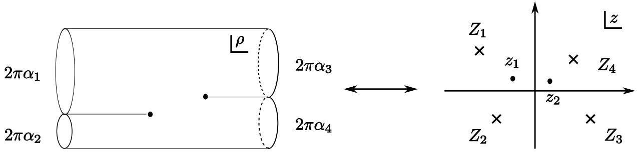
$$\times e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \prod_{I=1,2} \left(\partial^2\rho(z_I)\bar{\partial}^2\bar{\rho}(\bar{z}_I)\right)^{-\frac{3}{4}}$$

$e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$ is needed because $\hat{c} = d - 2$,.

For general d



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$e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$ behaves as $|z_1 - z_2|^{-\frac{d-2}{8}}$ in the limit $z_1 \to z_2$, and this amplitude is finite for large $-d$.

Motivation

- Dimensional regularization is possible.
- $X^{\pm}$ CFT $\longrightarrow$ the dimensional regularization preserves

  BRST on the worldsheet $\sim$ gauge symmetry of SF

- Using the CFT, one can show that the tree level (NS,NS) sector amplitudes derived from the SFT coincide with the results of the 1-st quantized formulation.

In collaboration with Y. Baba and K. Murakami (Riken)
arXiv:0906.3577 [hep-th] JHEP10(2009) 035
arXiv:0909.4675 [hep-th] JHEP to appear
arXiv:0911.3704 [hep-th]
arXiv:0912.****

Plan of the talk

1. $X^{\pm}$ CFT (bosonic)
2. Light-cone gauge amplitudes
3. $X^{\pm}$ CFT (super)
4. Dimensional regularization
5. Outlook

§1 $X^{\pm}$ CFT (bosonic)

We propose a 2d CFT with an action

$$S_{X^{\pm}} = -\frac{1}{2\pi}\int d^2z\left(\partial X^{+}\bar{\partial}X^{-}+\bar{\partial}X^{+}\partial X^{-}\right) + \frac{d-26}{24}\Gamma[\ln\left(\partial X^{+}\bar{\partial}X^{+}\right)]$$

Γ is the Liouville action

$$\Gamma\left[\phi\right] = -\frac{1}{\pi}\int d^2z\,\partial\phi\bar{\partial}\phi$$

- We calculate the correlation functions starting from this action.

$X^\pm$ CFT

energy momentum tensor

$$T_{X^\pm}(z) \equiv \partial X^+ \partial X^- - \frac{d-26}{12}\{X^+, z\}$$

Schwarzian derivative

$$\{X^+, z\} \equiv \frac{\partial^3 X^+}{\partial X^+} - \frac{3}{2}\left(\frac{\partial^2 X^+}{\partial X^+}\right)^2$$

From the correlation functions, one can see that the energy-momentum tensor satisfies the Virasoro algebra with $c = 28 - d$.

$X^{\pm}$ CFT

energy momentum tensor

$$T_{X^{\pm}}(z) \equiv \partial X^{+}\partial X^{-} - \frac{d-26}{12}\{X^{+}, z\}$$

Schwarzian derivative

$$\{X^{+}, z\} \equiv \frac{\partial^{3}X^{+}}{\partial X^{+}} - \frac{3}{2}\left(\frac{\partial^{2}X^{+}}{\partial X^{+}}\right)^{2}$$

Later, we will show that the tree amplitude of LC gauge SFT can be described by using this CFT.

Correlation functions

X^{+} should possess an nonzero expectation value.

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We always consider the theory in the presence of vertex operators $\exp\left(-ip^+X^-\right)$.

$$\left\langle F\left[X^+,X^-\right]\prod_{r=1}^{N}e^{-ip_r^+X^-}\left(Z_r,\bar{Z}_r\right)\right\rangle$$

$$\equiv\int\left[dX^+dX^-\right]e^{-S_{X^\pm}}F\left[X^+,X^-\right]\prod_{r=1}^{N}e^{-ip_r^+X^-}\left(Z_r,\bar{Z}_r\right)$$

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Let us calculate the correlation function for a functional $F\left[X^+\right]$ of X^+.

Correlation functions 1 $F\left[X^{+}\right]$

$$\left\langle F\left[X^{+}\right]\prod_{r=1}^{N}e^{-ip_r^{+}X^{-}}\left(Z_r,\bar{Z}_r\right)\right\rangle$$

$$= \int\left[dX^{\pm}\right]e^{-S_{X^{\pm}}}F\left[X^{+}\right]\prod_{r=1}^{N}e^{-ip_r^{+}X^{-}}\left(Z_r,\bar{Z}_r\right)$$

$$\begin{aligned} S_{X^{\pm}} &= -\frac{1}{2\pi}\int d^2z\left(\partial X^{+}\bar{\partial}X^{-}+\bar{\partial}X^{+}\partial X^{-}\right) \\ &\quad + \frac{d-26}{24}\Gamma[\ln\left(\partial X^{+}\bar{\partial}X^{+}\right)] \end{aligned}$$

This should be considered as a Euclideanized version of a Lorentzian path integral.

Correlation functions 1 $F\left[X^{+}\right]$

$$\left\langle F\left[X^{+}\right]\prod_{r=1}^{N}e^{-ip_{r}^{+}X^{-}}\left(Z_{r},\bar{Z}_{r}\right)\right\rangle$$

$$=\int\left[dX\right]\exp\left(\frac{1}{\pi}\int d^{2}zX^{-}\left(\partial\bar{\partial}X^{+}-\pi i\sum_{r=1}^{N}p_{r}^{+}\delta^{2}\left(z-Z_{r}\right)\right)\right)$$
$$\times F\left[X^{+}\right]\exp\left(-\frac{d-26}{24}\Gamma\left[\ln\left(\partial X^{+}\bar{\partial}X^{+}\right)\right]\right)$$

$$-\frac{i}{2}\partial\bar{\partial}\left(\rho\left(z\right)+\bar{\rho}\left(\bar{z}\right)\right)\ =\ \pi i\sum_{r=1}^{N}p_{r}^{+}\delta^{2}\left(z-Z_{r}\right)$$

Correlation functions 1 $F\left[X^{+}\right]$

$$\left\langle F\left[X^{+}\right]\prod_{r=1}^{N}e^{-ip_r^{+}X^{-}}\left(Z_r,\bar{Z}_r\right)\right\rangle$$

$$\sim\left(\det\left(\partial\bar{\partial}\right)\right)^{-1}F\left[-\frac{i}{2}\left(\rho+\bar{\rho}\right)\right]\exp\left(-\frac{d-26}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]\right)$$

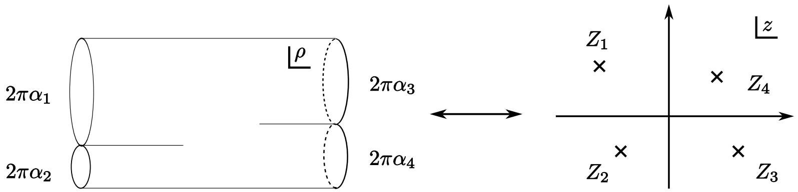
$$\rho\left(z\right)=\sum_{r=1}^{N}\alpha_r\ln\left(z-Z_r\right)\quad\left(\alpha_r\equiv 2p_r^{+}\right)$$

Mandelstam mapping

This implies that X^+ has an expectation value

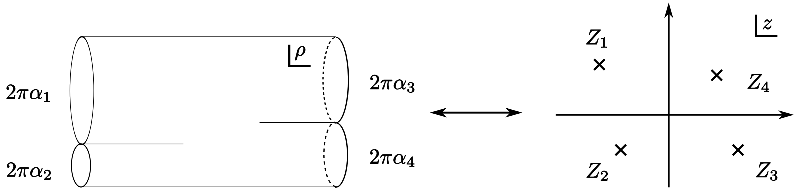
$$\left\langle X^+(z,\bar{z})\right\rangle \sim -\frac{i}{2}\left(\rho(z)+\bar{\rho}(\bar{z})\right)$$
$$\rho(z) = \sum_{r=1}^{N}\alpha_r \ln(z-Z_r) \quad \left(\alpha_r \equiv 2p_r^+\right)$$

$\rho(z)$ coincides with the Mandelstam mapping.



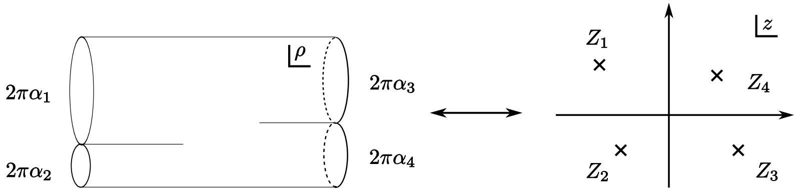
Remarks

1. $\left\langle F\left[X^{+}\right] \prod_{r=1}^{N} e^{-ip_r^{+}X^{-}}\left(Z_r, \bar{Z}_r\right)\right\rangle$ is used to express the tree amplitude corresponding to the light-cone diagram.



Remarks

2.$e^{-ip_r^+ X^-}(Z_r, \bar{Z}_r)$ corresponds to a hole with length $2\pi\alpha_r$, similar to the macroscopic loop oprator in the old matrix models.



Correlation functions 2 X^{-} insertions

Correlation functions with X^{-} insertions can be calculated using $\left\langle F\left[X^{+}\right]\prod_{r=1}^{N} e^{-ip_r^+X^-}\left(Z_r,\bar{Z}_r\right)\right\rangle$ as a generating functional.

$$\left\langle F\left[X^{+}\right]X^{-}(Z_N,\bar{Z}_N)\prod_{r=1}^{N-1} e^{-ip_r^+X^-}\left(Z_r,\bar{Z}_r\right)\right\rangle$$

$$\sim i\partial_{p_N^+}\left\langle F\left[X^{+}\right]\prod_{r=1}^{N} e^{-ip_r^+X^-}\left(Z_r,\bar{Z}_r\right)\right\rangle\Bigg|_{p_N^+=0}$$

$$\sim i\partial_{p_N^+}\left(F\left[-\frac{i}{2}\left(\rho+\bar{\rho}\right)\right]\exp\left(-\frac{d-26}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]\right)\right)\Bigg|_{p_N^+=0}$$

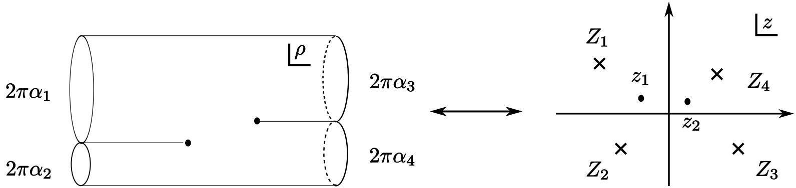
Evaluation of $\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]$

One can derive all the correlation functions from $\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]$.

$$\Gamma[\phi] = -\frac{1}{\pi}\int d^2z\,\partial\phi\bar{\partial}\phi$$

$$\rho(z) = \sum_{r=1}^{N}\alpha_r\ln\left(z-Z_r\right)$$

$\partial\rho(z)$ has poles at $z\sim Z_r$ and zeros at $z\sim z_I$.

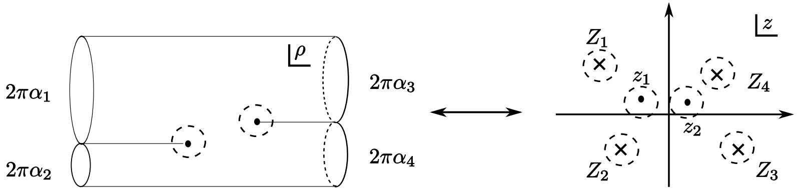


$$ds^2 = d\rho d\bar{\rho} = \partial\rho\bar{\partial}\bar{\rho}dzd\bar{z}$$

Evaluation of $\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]$

There are at least two ways to obtain $\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]$.

1. Direct evaluation regularizing the singularities
(Mandelstam, lectures at "Unified String Theory")



2. Integration of the variation $\delta\Gamma$ under $T \to T + \delta T$
(Cremmer and Gervais, Baba, Murakami and N.I.)

Evaluation of $\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]$

$$\exp\left(-\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]\right) = e^{-W} e^{-2\sum_{r=1}^{N} \mathrm{Re}\, \bar{N}_{00}^{rr}} \prod_{I} \left|\partial^2\rho(z_I)\right|^{-3}$$

$$e^{-W} \equiv \frac{\prod_{I>J}|z_I - z_J|^4 \prod_{r>s}|Z_r - Z_s|^4}{\prod_{r,I}|Z_r - z_I|^4}$$

$$\bar{N}_{00}^{rr} \equiv \frac{\rho(z_I)}{\alpha_r} - \sum_{s\neq r} \frac{\alpha_s}{\alpha_r} \ln\left(Z_r - Z_s\right)$$

Evaluation of $\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]$

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$$= e^{-2\sum_{r=1}^{N} \mathrm{Re}\,\bar{N}_{00}^{rr}} \times \frac{\left|\sum_{s=1}^{N}\alpha_s Z_s\right|^{-2N+6} \prod_{r>s}\left|Z_r - Z_s\right|^2}{\prod_{r=1}^{N}\left|\alpha_r\right| \prod_{I>J}\left|z_I - z_J\right|^2}$$

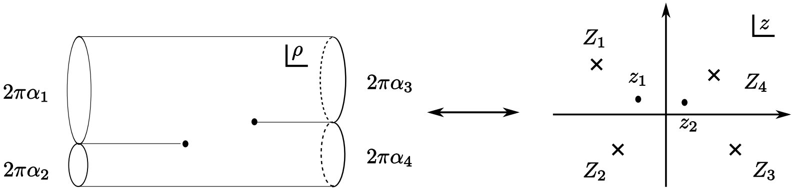
$\exp\left(-\frac{d-2}{24}\Gamma\left(\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right)\right) \sim |z_I - z_J|^{-\frac{d-2}{12}}$ for $z_I \to z_J$

Energy-momentum tensor

From the correlation functions of the energy-momentum tensor, we can deduce

- $T_{X^\pm}(z)$ is regular at the points where there are no operator insertions.
- OPE

$$T_{X^\pm}(z)e^{-ip_r^+X^-}(Z_r,\bar{Z}_r) \sim \frac{1}{z-Z_r}\partial e^{-ip_r^+X^-}(Z_r,\bar{Z}_r)$$



$$ds^2 = d\rho d\bar{\rho} = \partial\rho\bar{\partial}\bar{\rho}dzd\bar{z}$$

Energy-momentum tensor

OPE

$$\begin{aligned}
\partial X^{+}(z)\partial X^{+}(z') &\sim \text{regular} \\
\partial X^{-}(z)\partial X^{+}(z') &\sim \frac{1}{(z-z')^2} \\
\partial X^{-}(z)\partial X^{-}(z') &\sim -\frac{d-26}{12}\partial_z\partial_{z'}\left[\frac{1}{(z-z')^2}\frac{1}{\partial X^{+}(z)\partial X^{+}(z')}\right]
\end{aligned}$$

Energy-momentum tensor

OPE

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\partial X^+(z)\partial X^+(z') &\sim \text{regular} \\
\partial X^-(z)\partial X^+(z') &\sim \frac{1}{(z-z')^2} \\
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\end{aligned}$$

From these, one can deduce

$$T_{X^\pm}(z)T_{X^\pm}(z') \sim \frac{\frac{1}{2}(28-d)}{(z-z')^4}+\frac{2}{(z-z')^2}T_{X^\pm}(z')+\frac{1}{z-z'}\partial T_{X^\pm}(z')$$

Energy-momentum tensor

OPE

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$T_{X^{\pm}}$ satisfies the Virasoro algebra with $c = 28 - d$

§2 Light-cone gauge amplitudes

We would like to show that the X^+ CFT can be used to describe LC string theory in $d \neq 26$ dimensions.

- We consider bosonic closed string field theory for $d \neq 26$.
- We show that the tree amplitude can be written in a BRST invariant form using the $X^{\pm}$ CFT.

String field action for $d \neq 26$

action

$$S = \int dt \left[\frac{1}{2} \int d1d2 \left\langle R\left(1,2\right)\right| \Phi\rangle_1 \left(i\frac{\partial}{\partial t} - H \right) |\Phi\rangle_2 + \frac{2g}{3} \int d1d2d3 \left\langle V_3\left(1,2,3\right)\right| \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \right]$$

String field action for $d \neq 26$

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$$dr = \frac{\alpha_r d\alpha_r}{4\pi} \frac{d^{d-2}p_r}{(2\pi)^{d-2}}$$

$$\langle R(1,2)| = \frac{1}{\alpha_1} 4\pi\delta(\alpha_1+\alpha_2)(2\pi)^{d-2}\delta(p_1+p_2)\,{}_{12}\langle 0| e^{E(1,2)}$$

$$E(1,2) = -\sum_{n=1}^{\infty} \frac{1}{n} \left(\alpha_n^{i(1)} \alpha_n^{i(2)} + \tilde{\alpha}_n^{i(1)} \tilde{\alpha}_n^{i(2)} \right)$$

$$H = \frac{L_0^{\mathrm{LC}(2)} + \tilde{L}_0^{\mathrm{LC}(2)} - \frac{d-2}{12}}{\alpha_2}$$

String field action for $d \neq 26$

action

$$S = \int dt \left[\frac{1}{2} \int d1d2 \left\langle R(1,2) \right| \Phi\rangle_1 \left(i\frac{\partial}{\partial t} - H \right) |\Phi\rangle_2 + \frac{2g}{3} \int d1d2d3 \left\langle V_3(1,2,3) \right| \Phi\rangle_1 |\Phi\rangle_2 |\Phi\rangle_3 \right]$$

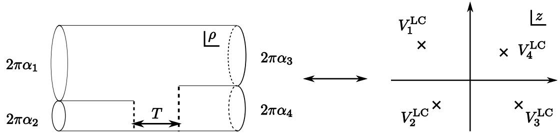
$$\langle V_3(1,2,3)| = 4\pi\delta\left(\sum_{r=1}^{3}\alpha_r\right)(2\pi)^{d-2}\delta^{d-2}\left(\sum_{r=1}^{3}p_r\right) \times \left\langle V_3^{\mathrm{LPP}}(1,2,3)\right| e^{-\Gamma^{[3]}(1,2,3)}$$

$$e^{-\Gamma^{[3]}(1,2,3)} = \mathrm{sgn}\left(\alpha_1\alpha_2\alpha_3\right)\left|\frac{e^{-2\hat{\tau}_0\sum_r\frac{1}{\alpha_r}}}{\alpha_1\alpha_2\alpha_3}\right|^{\frac{d-2}{24}}$$

Light-cone gauge amplitudes

With this choice of the action, tree amplitudes in the LC gauge SFT is given as

$$\mathcal{A} \sim \int \prod_I d^2\mathcal{T}_I \left\langle \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{X^i} e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$$



$$ds^2 = d\rho d\bar{\rho} = \partial\rho\bar{\partial}\bar{\rho}dzd\bar{z}$$

- We would like to recast the amplitude into a BRST invariant form using the $X^{\pm}$ CFT.

$X^{\pm}$ and ghost

$$
\begin{aligned}
&e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\quad = e^{-\frac{d-24}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\qquad \times \prod_{r=1}^{N} |\alpha_r|^{-1} \, e^{-\frac{1}{2}W} \left| \sum_{s=1}^{N} \alpha_s Z_s \right|^2 e^{-2\sum_{r=1}^{N} \mathrm{Re}\,\bar{N}_{00}^{rr}} \prod_I \left|\partial^2\rho(z_I)\right|^{-2}
\end{aligned}
$$

$X^\pm$ and ghost

$$
\begin{aligned}
& e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
& \quad = e^{-\frac{d-24}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
& \qquad \times \prod_{r=1}^{N} \left|\alpha_r\right|^{-1} e^{-\frac{1}{2}W} \left|\sum_{s=1}^{N} \alpha_s Z_s\right|^2 e^{-2\sum_{r=1}^{N} \mathrm{Re}\,\bar{N}_{00}^{rr}} \prod_I \left|\partial^2\rho(z_I)\right|^{-2}
\end{aligned}
$$

$$
\begin{aligned}
e^{-\frac{1}{2}W} &= \frac{\prod_{I>J}\left|z_I - z_J\right|^2 \prod_{r>s}\left|Z_r - Z_s\right|^2}{\prod_{r,I}\left|Z_r - z_I\right|^2} \\
&\sim \int \left[d\left(\mathrm{ghost}\right)\right] e^{-S_{bc}} \left(\lim_{z\to\infty} \frac{1}{|z|^4} c\left(z\right)\tilde{c}\left(\bar{z}\right)\right) \\
&\quad \times \prod_I \left(b(z_I)\tilde{b}(\bar{z}_I)\right) \prod_r \left(c\left(Z_r\right)\tilde{c}\left(\bar{Z}_r\right)\right)
\end{aligned}
$$

$X^\pm$ and ghost

$$
\begin{aligned}
& e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\quad = e^{-\frac{d-24}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\qquad \times \prod_{r=1}^{N} |\alpha_r|^{-1}\, e^{-\frac{1}{2}W} \left|\sum_{s=1}^{N}\alpha_s Z_s\right|^2 e^{-2\sum_{r=1}^{N}\mathrm{Re}\,\bar{N}_{00}^{rr}} \prod_I \left|\partial^2\rho(z_I)\right|^{-2}
\end{aligned}
$$

$$
e^{-\frac{d-24}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \sim \int \left[dX^\pm\right] e^{-S_{X^\pm}} \prod_{r=1}^{N} e^{-ip_r^+ X^-}\left(Z_r, \bar{Z}_r\right)
$$

$X^{\pm}$ and ghost

$$
\begin{aligned}
& e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\quad = e^{-\frac{d-24}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\qquad \times \prod_{r=1}^{N} \left|\alpha_r\right|^{-1} e^{-\frac{1}{2}W} \left|\sum_{s=1}^{N}\alpha_s Z_s\right|^2 e^{-2\sum_{r=1}^{N}\mathrm{Re}\,\bar{N}_{00}^{rr}} \prod_I \left|\partial^2\rho(z_I)\right|^{-2}
\end{aligned}
$$

$$
\begin{aligned}
& \sim \int \left[dX^{\pm}\right] e^{-S_{X^{\pm}}} \prod_{r=1}^{N}\left(e^{-ip_r^{+}X^{-}}\left(Z_r,\bar{Z}_r\right)\left|\alpha_r\right|^{-1} e^{-2\,\mathrm{Re}\,\bar{N}_{00}^{rr}}\right) \\
&\quad \times \int \left[d\left(\text{ghost}\right)\right] e^{-S_{bc}} \left|\sum_r \alpha_r Z_r\right|^2 \left(\lim_{z\to\infty}\frac{1}{|z|^4}c\left(z\right)\tilde{c}\left(\bar{z}\right)\right) \\
&\qquad \times \prod_I \left(\frac{b}{\partial^2\rho}\left(z_I\right)\frac{\tilde{b}}{\bar{\partial}^2\bar{\rho}}\left(\bar{z}_I\right)\right) \prod_r \left(c\left(Z_r\right)\tilde{c}\left(\bar{Z}_r\right)\right)
\end{aligned}
$$

$X^{\pm}$ and ghost

Substituting this, we obtain

$$\left\langle \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{X^i} e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$$

$$\sim \int \left[dX^{\mu} d\left(\mathrm{ghost}\right)\right] e^{-S_X - S_{bc}} \left|\sum_r \alpha_r Z_r\right|^2 \left(\lim_{z\to\infty} \frac{1}{|z|^4} c\left(z\right)\tilde{c}\left(\bar{z}\right)\right)$$
$$\times \prod_I \left(\frac{b}{\partial^2\rho}(z_I)\frac{\tilde{b}}{\bar{\partial}^2\bar{\rho}}(\bar{z}_I)\right)$$
$$\times \prod_{r=1}^{N} \left(c\tilde{c}\frac{V_r^{\mathrm{LC}}}{\alpha_r} e^{-ip_r^+ X^- - 2\,\mathrm{Re}\,\bar{N}_{00}^{rr}}\right)\left(Z_r, \bar{Z}_r\right)$$

$X^{\pm}$ and ghost

Substituting this, we obtain

$$\left\langle \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{X^i} e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$$

$$\sim \left|\sum_r \alpha_r Z_r\right|^2 \left\langle \left(\lim_{z\to\infty} \frac{1}{|z|^4} c(z)\,\tilde{c}(\bar{z})\right) \prod_I \left(\frac{b}{\partial^2\rho}(z_I)\frac{\tilde{b}}{\bar{\partial}^2\bar{\rho}}(\bar{z}_I)\right) \right.$$
$$\left. \times \prod_{r=1}^{N} \left(c\tilde{c}\frac{V_r^{\mathrm{LC}}}{\alpha_r} e^{-ip_r^+ X^- - 2\,\mathrm{Re}\,\bar{N}_{00}^{rr}}\right)(Z_r, \bar{Z}_r) \right\rangle_{X^\mu, b, c}$$

BRST invariant form

$$
\begin{aligned}
\mathcal{A} &= \int \prod_I d^2\mathcal{T}_I \left\langle \prod_{r=1}^N V_r^{\mathrm{LC}} \right\rangle_{X^i} e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\sim \int \prod_I d^2\mathcal{T}_I \left| \sum_r \alpha_r Z_r \right|^2 \\
&\quad \times \left\langle \left(\lim_{z\to\infty} \frac{1}{|z|^4} c(z)\,\tilde{c}(\bar{z}) \right) \prod_I \left(\frac{b}{\partial^2\rho}(z_I) \frac{\tilde{b}}{\bar{\partial}^2\bar{\rho}}(\bar{z}_I) \right) \right. \\
&\quad\quad \left. \times \prod_{r=1}^N \left(c\tilde{c} \frac{V_r^{\mathrm{LC}}}{\alpha_r} e^{-ip_r^+ X^- - 2\,\mathrm{Re}\,\bar{N}_{00}^{rr}} \right) (Z_r, \bar{Z}_r) \right\rangle_{X^\mu, b, c}
\end{aligned}
$$

BRST invariant form

$$
\begin{aligned}
\mathcal{A} &= \int \prod_I d^2\mathcal{T}_I \left\langle \prod_{r=1}^N V_r^{\mathrm{LC}} \right\rangle_{X^i} e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\sim \int \prod_I d^2\mathcal{T}_I \left| \sum_r \alpha_r Z_r \right|^2 \\
&\quad \times \left\langle \left(\lim_{z\to\infty} \frac{1}{|z|^4} c(z)\,\tilde{c}(\bar{z}) \right) \prod_I \left(\frac{b}{\partial^2\rho}(z_I) \frac{\tilde{b}}{\bar{\partial}^2\bar{\rho}}(\bar{z}_I) \right) \right. \\
&\quad \left. \times \prod_{r=1}^N \left(c\tilde{c} \frac{V_r^{\mathrm{LC}}}{\alpha_r} e^{-ip_r^+ X^- - 2\,\mathrm{Re}\,\bar{N}_{00}^{rr}} \right)(Z_r, \bar{Z}_r) \right\rangle_{X^\mu, b, c}
\end{aligned}
$$

Replacing $\rho + \bar{\rho}$ by $2iX^+$ in the braces

BRST invariant form

$$
\begin{aligned}
\mathcal{A} &= \int \prod_I d^2\mathcal{T}_I \left\langle \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{X^i} e^{-\frac{d-2}{24}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]} \\
&\sim \int \prod_I d^2\mathcal{T}_I \left| \sum_r \alpha_r Z_r \right|^2 \\
&\quad \times \left\langle \left(\lim_{z\to\infty} \frac{1}{|z|^4} c(z)\,\tilde{c}(\bar{z}) \right) \prod_I \left(\frac{b}{\partial^2\rho}(z_I) \frac{\tilde{b}}{\bar{\partial}^2\bar{\rho}}(\bar{z}_I) \right) \right. \\
&\qquad \left. \times \prod_{r=1}^{N} \left(c\tilde{c} \frac{V_r^{\mathrm{LC}}}{\alpha_r} e^{-ip_r^+ X^- - 2\,\mathrm{Re}\,\bar{N}_{00}^{rr}} \right) (Z_r, \bar{Z}_r) \right\rangle_{X^\mu, b, c}
\end{aligned}
$$

rewriting $\frac{b}{\partial^2\rho}(z_I)$ as $\oint_{z_I} \frac{dz}{2\pi i} \frac{b}{\partial\rho}(z)$ and deforming the contour we obtain

BRST invariant form

$$\mathcal{A} \sim \prod_{r=4}^{N} \int d^2 Z_r \left\langle \prod_{r=1}^{3} \left(c\tilde{c} V_r^{\prime \mathrm{DDF}} \right) (Z_r, \bar{Z}_r) \prod_{r=4}^{N} V_r^{\prime \mathrm{DDF}} (Z_r, \bar{Z}_r) \right.$$

$$\left. \times \prod_{r=1}^{N} \exp \left(i \frac{d-26}{24} \frac{X^+}{p_r^+} \right) \left(z_I^{(r)}, \bar{z}_I^{(r)} \right) \right\rangle$$

BRST invariant form

$$\mathcal{A} \sim \prod_{r=4}^{N} \int d^2 Z_r \left\langle \prod_{r=1}^{3} \left(c\tilde{c}V_r^{\prime\mathrm{DDF}}\right)(Z_r, \bar{Z}_r) \prod_{r=4}^{N} V_r^{\prime\mathrm{DDF}}(Z_r, \bar{Z}_r) \right.$$
$$\left. \times \prod_{r=1}^{N} \exp\left(i\frac{d-26}{24}\frac{X^+}{p_r^+}\right)\left(z_I^{(r)}, \bar{z}_I^{(r)}\right) \right\rangle$$

$$V_r^{\prime\mathrm{DDF}} \equiv \; : V_r^{\mathrm{DDF}} \exp\left(-i\frac{d-26}{24}\frac{X^+}{p_r^+}\right) :$$

V_r^{DDF} is the DDF operator which corresponds to V_r^{LC}.

BRST invariant form

$$\mathcal{A} \sim \prod_{r=4}^{N} \int d^2 Z_r \left\langle \prod_{r=1}^{3} \left(c\tilde{c}V_r^{\prime\mathrm{DDF}}\right)(Z_r, \bar{Z}_r) \prod_{r=4}^{N} V_r^{\prime\mathrm{DDF}}(Z_r, \bar{Z}_r) \right.$$
$$\left. \times \prod_{r=1}^{N} \exp\left(i\frac{d-26}{24}\frac{X^+}{p_r^+}\right)\left(z_I^{(r)}, \bar{z}_I^{(r)}\right) \right\rangle$$

V_r^{DDF} : primary field of weight $\left(\frac{d-2}{24}, \frac{d-2}{24}\right)$

$V_r^{\prime\mathrm{DDF}}$: primary field of weight $(1,1)$

BRST invariant form

$$\mathcal{A} \sim \prod_{r=4}^{N} \int d^2 Z_r \left\langle \prod_{r=1}^{3} \left(c\tilde{c} V_r'^{\mathrm{DDF}} \right) (Z_r, \bar{Z}_r) \prod_{r=4}^{N} V_r'^{\mathrm{DDF}} (Z_r, \bar{Z}_r) \right.$$
$$\left. \times \prod_{r=1}^{N} \exp\left(i \frac{d-26}{24} \frac{X^+}{p_r^+} \right) \left(z_I^{(r)}, \bar{z}_I^{(r)} \right) \right\rangle$$

$\exp\left(i \frac{d-26}{24} \frac{X^+}{p_r^+} \right) \left(z_I^{(r)}, \bar{z}_I^{(r)} \right)$ commutes with $T_{X^{\pm}}, Q_{\mathrm{B}}$.

BRST invariant form

$$\mathcal{A} \sim \prod_{r=4}^{N} \int d^2 Z_r \left\langle \prod_{r=1}^{3} \left(c\tilde{c}V_r^{\prime\mathrm{DDF}}\right)(Z_r, \bar{Z}_r) \prod_{r=4}^{N} V_r^{\prime\mathrm{DDF}}(Z_r, \bar{Z}_r) \right.$$
$$\left. \times \prod_{r=1}^{N} \exp\left(i\frac{d-26}{24} \frac{X^+}{p_r^+} \right) \left(z_I^{(r)}, \bar{z}_I^{(r)} \right) \right\rangle$$

This form of the amplitude is BRST invariant.

BRST invariant form

$$\mathcal{A} \sim \prod_{r=4}^{N} \int d^2 Z_r \left\langle \prod_{r=1}^{3} \left(c\tilde{c} V_r'^{\mathrm{DDF}}\right)(Z_r, \bar{Z}_r) \prod_{r=4}^{N} V_r'^{\mathrm{DDF}} (Z_r, \bar{Z}_r) \times \prod_{r=1}^{N} \exp\left(i \frac{d-26}{24} \frac{X^+}{p_r^+} \right) \left(z_I^{(r)}, \bar{z}_I^{(r)} \right) \right\rangle$$

The LC gauge SFT in d dimensions is described by the worldsheet theory

$$X^i + \text{ghost} + X^{\pm}\text{CFT}$$

§3 $X^{\pm}$ CFT (super)

Let us consider the supersymmetric generalization of the results so far.

superspace coordinate

$$\mathbf{z} = (z, \theta)$$

superfield

$$X^{\pm}(\mathbf{z}, \bar{\mathbf{z}}) = x^{\pm} + i\theta\psi^{\pm} + i\bar{\theta}\bar{\psi}^{\pm} + i\theta\bar{\theta}F^{\pm}$$

covariant derivative

$$D = \partial_{\theta} + \theta\partial_z$$

It is convenient to introduce

$$\Theta^{+}(\mathbf{z}) = \frac{DX^{+}}{(\partial X^{+})^{\frac{1}{2}}}(\mathbf{z})$$

so that the map $\mathbf{z} = (z, \theta) \mapsto \mathbf{X}_L^{+}(\mathbf{z}) = \left(X_L^{+}(\mathbf{z}), \Theta^{+}(\mathbf{z})\right)$ is a superconformal mapping.

$X^{\pm}$ CFT (super)

We propose a superconformal field theory with an action

$$S_{X^{\pm}} = -\frac{1}{2\pi}\int d^2\mathbf{z}\left(\bar{D}X^{+}DX^{-}+\bar{D}X^{-}DX^{+}\right)+\frac{d-10}{8}\Gamma_{\text{super}}\left[\Phi\right]$$

$\Gamma_{\text{super}}\left[\Phi\right]$ is the super Liouville action

$$\begin{aligned}\Gamma_{\text{super}}\left[\Phi\right] &= -\frac{1}{2\pi}\int d^2\mathbf{z}\bar{D}\Phi D\Phi \\ \Phi\left(\mathbf{z},\bar{\mathbf{z}}\right) &= \ln\left(-4\left(D\Theta^{+}\right)^2(\mathbf{z})\left(\bar{D}\bar{\Theta}^{+}\right)^2(\bar{\mathbf{z}})\right)\end{aligned}$$

- We calculate the correlation functions starting from this action.

$X^{\pm}$ CFT (super)

energy-momentum tensor

$$T_{X^{\pm}}(\mathbf{z}) = \frac{1}{2}DX^{+}\partial X^{-} + \frac{1}{2}DX^{-}\partial X^{+} - \frac{d-10}{4}S(\mathbf{z},\mathbf{X}_L^{+})$$

super Schwarzian derivative

$$S(\mathbf{z},\mathbf{X}_L^{+}) = \frac{D^4\Theta^{+}}{D\Theta^{+}} - 2\frac{D^3\Theta^{+}D^2\Theta^{+}}{(D\Theta^{+})^2}$$

- From the correlation functions, one can see that the energy-momentum tensor satisfies the Virasoro algebra with $\hat{c} = 12 - d$.

Correlation functions

Calculations are essentially the same as those in the bosonic case.

$$\left\langle F\left[X^{+}\right]\prod_{r=1}^{N}e^{-ip_r^{+}X^{-}}\left(\mathbf{Z}_r,\bar{\mathbf{Z}}_r\right)\right\rangle$$

$$\sim F\left[-\frac{i}{2}\left(\rho+\bar{\rho}\right)\right]\exp\left(-\frac{d-10}{8}\Gamma_{\text{super}}\left[\ln\left(\left(D\xi\right)^{2}\left(\bar{D}\bar{\xi}\right)^{2}\right)\right]\right)$$

super Mandelstam mapping

$$\rho\left(\mathbf{z}\right)=\sum_{r=1}^{N}\alpha_r\ln\left(\mathbf{z}-\mathbf{Z}_r\right)$$

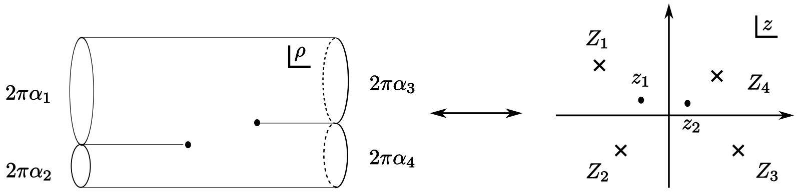
$$\xi\left(\mathbf{z}\right)=\frac{D\rho}{\left(\partial\rho\right)^{\frac{1}{2}}}\left(\mathbf{z}\right)$$

where

$$\mathbf{z}-\mathbf{z}'=z-z'-\theta\theta'$$

Evaluation of Γ_{super}

Much more complicated compared with the bosonic case.



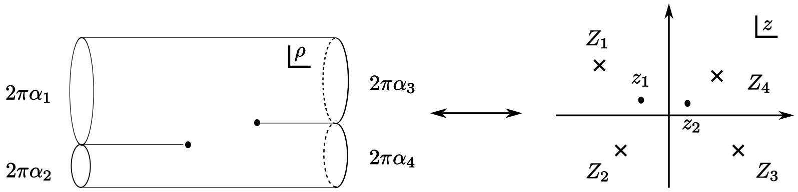
interaction points $\mathbf{z}_I$

$$\partial\rho(\mathbf{z}_I) - \frac{1}{2}\frac{\partial^2 D\rho D\rho}{\partial^2\rho}(\mathbf{z}_I) = 0\ , \qquad \partial D\rho(\mathbf{z}_I) - \frac{1}{6}\frac{\partial^3\rho D\rho}{\partial^2\rho}(\mathbf{z}_I) = 0$$

$\mathbf{z}_I$ is different from the naive generalization $\tilde{\mathbf{z}}_I$ satisfying $\partial\rho\,(\tilde{\mathbf{z}}_I) = 0\ , \partial D\rho\,(\tilde{\mathbf{z}}_I) = 0$.

Evaluation of Γ_{super}

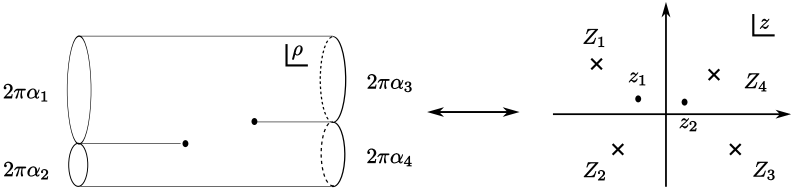
Much more complicated compared with the bosonic case.



This is because of the presence of the odd supermoduli $\xi_I = \frac{D\rho}{(\partial^2\rho)^{\frac{1}{4}}}(\mathbf{z}_I) \neq 0$.

Evaluation of Γ_{super}

Much more complicated compared with the bosonic case.



Although it is very complicated, it is possible to evaluate Γ_{super}.
(Berkovits, Baba-Murakami-N.I.)

Evaluation of $\Gamma_{\rm super}$

$$
\begin{aligned}
-\Gamma_{\rm super} \quad = \quad & -W_{\rm super} - \frac{1}{2}\sum_r \bar{N}^{rr}_{00} \\
& - \frac{3}{4}\sum_I \ln\left(\partial^2\rho - \frac{13}{9}\frac{\partial^3 D\rho D\rho}{\partial^2\rho} + \frac{8}{3}\frac{\partial^3\rho\partial^2 D\rho D\rho}{(\partial^2\rho)^2}\right)(\tilde{\mathbf{z}}_I) \\
& + \text{c.c.} \ .
\end{aligned}
$$

$$
\begin{aligned}
-W_{\rm super} \quad \equiv \quad & \sum_{r>s}\ln(\mathbf{Z}_r - \mathbf{Z}_s) + \sum_{I>J} P_I P_J \ln(\tilde{\mathbf{z}}_I - \tilde{\mathbf{z}}_J) \\
& - \sum_r \sum_I P_I \ln(\mathbf{Z}_r - \tilde{\mathbf{z}}_I)
\end{aligned}
$$

$$
P_I \quad \equiv \quad 1 + \frac{\partial^2 D\rho D\rho}{(\partial^2\rho)^2}(\tilde{\mathbf{z}}_I)\,\tilde{\partial}_I + \frac{D\rho}{\partial^2\rho}(\tilde{\mathbf{z}}_I)\,\tilde{\partial}_I\tilde{D}_I
$$

Evaluation of $\Gamma_{\rm super}$

$$
\begin{aligned}
-\Gamma_{\rm super} \quad = \quad & -W_{\rm super} - \frac{1}{2}\sum_r \bar{N}_{00}^{rr} \\
& - \frac{3}{4}\sum_I \ln\left(\partial^2\rho - \frac{13}{9}\frac{\partial^3 D\rho D\rho}{\partial^2\rho} + \frac{8}{3}\frac{\partial^3\rho\partial^2 D\rho D\rho}{(\partial^2\rho)^2}\right)(\tilde{\mathbf{z}}_I) \\
& + \text{c.c.}\ .
\end{aligned}
$$

$$
\bar{N}_{00}^{rr} \quad \equiv \quad \frac{\rho(\tilde{\mathbf{z}}_I^{(r)})}{\alpha_r} - \sum_{s\neq r}\frac{\alpha_s}{\alpha_r}\ln\left(\mathbf{Z}_r - \mathbf{Z}_s\right)
$$

Energy-momentum tensor

In principle, one can evaluate all the correlation functions starting from Γ_{super}.

From the correlation functions, we can see

- $T_{X^{\pm}}$ is regular at the points where no operators are inserted.
- OPE

$$
\begin{aligned}
&T_{X^{\pm}}(\mathbf{z})e^{-ip_r^+X^-}(\mathbf{Z}_r,\bar{\mathbf{Z}}_r) \\
&\quad\sim \frac{1}{\mathbf{z}-\mathbf{Z}_r}\frac{1}{2}De^{-ip_r^+X^-}(\mathbf{Z}_r,\bar{\mathbf{Z}}_r)+\frac{\theta-\Theta_r}{\mathbf{z}-\mathbf{Z}_r}\partial e^{-ip_r^+X^-}(\mathbf{Z}_r,\bar{\mathbf{Z}}_r)
\end{aligned}
$$

Energy-momentum tensor

$T_{X^{\pm}}$ satisfies super Virasoro algebra with $\hat{c} = 12 - d$

$$T_{X^{\pm}}(\mathbf{z})\, T_{X^{\pm}}(\mathbf{z}') \sim \frac{12-d}{4\left(\mathbf{z}-\mathbf{z}'\right)^3} + \frac{\theta-\theta'}{\left(\mathbf{z}-\mathbf{z}'\right)^2}\frac{3}{2}T_{X^{\pm}}(\mathbf{z}') + \frac{1}{\mathbf{z}-\mathbf{z}'}\frac{1}{2}DT_{X^{\pm}}(\mathbf{z}') + \frac{\theta-\theta'}{\mathbf{z}-\mathbf{z}'}\partial T_{X^{\pm}}(\mathbf{z}')$$

With the ghost superfield $B(\mathbf{z}) = \beta(z) + \theta b(z)$, $C(\mathbf{z}) = c(z) + \theta\gamma(z)$ and the transverse variables $X^i(\mathbf{z},\bar{\mathbf{z}})$, one can construct a nilpotent BRST charge

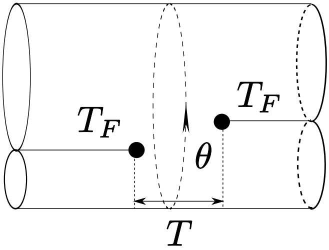
$$Q_{\mathrm{B}} = \oint \frac{d\mathbf{z}}{2\pi i}\left[-C\left(T_{X^{\pm}} - \frac{1}{2}DX^i\partial X^i\right) + \left(C\partial C - \frac{1}{4}(DC)^2\right)B\right]$$

§4 Dimensional regularization

Tree amplitudes for superstrings with (NS,NS) external lines

$$\mathcal{A} \sim \int \prod_I d^2\mathcal{T}_I \left\langle \prod_I \left[T_F^{\mathrm{LC}}(z_I)\,\tilde{T}_F^{\mathrm{LC}}(\bar{z}_I)\right] \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle_{X^i,\psi^i}$$
$$\times \prod_I \left(\partial^2\rho(z_I)\,\bar{\partial}^2\bar{\rho}(\bar{z}_I)\right)^{-\frac{3}{4}} e^{-\frac{d-2}{16}\Gamma\left[\ln\left(\partial\rho\bar{\partial}\bar{\rho}\right)\right]}$$

$$\mathcal{A} \sim \int dT d\theta$$



Dimensional regularization

Longitudinal variables and ghosts are introduced and

$$\mathcal{A}_N \sim \left\langle \prod_{r=1}^{3}\left[c\tilde{c}e^{-\phi-\tilde{\phi}}V_r^{\prime\mathrm{DDF}}\left(Z_r,\bar{Z}_r\right)\right] \times\prod_{r=4}^{N}\int d^2Z_r\prod_{r=4}^{N}e^{-\phi-\tilde{\phi}}V_r^{\prime\mathrm{DDF}}\left(Z_r,\bar{Z}_r\right) \times\prod_{I}\left[X(z_I)\tilde{X}(\bar{z}_I)\right]\prod_{r=1}^{N}e^{\frac{d-10}{16}\frac{i}{p_r^+}X^+}\left(z_I^{(r)},\bar{z}_I^{(r)}\right)\right\rangle$$

- $X\left(z\right)=\left\{Q_{\mathrm{B}}\,,\,\xi(z)\right\}$: the picture changing operator
- $V_r^{\prime\mathrm{DDF}}=:V_r^{\mathrm{DDF}}e^{-\frac{d-10}{16}\frac{i}{p_r^+}X^+}$: a superconformal primary with weight $\left(\frac{1}{2},\frac{1}{2}\right)$

Dimensional regularization

Everything is BRST invariant. By standard procedure

$$\mathcal{A}_N \sim \Bigg\langle \prod_{r=1}^{2}\left[c\tilde{c}e^{-\phi-\tilde{\phi}}V_r^{\prime\mathrm{DDF}}\right] \times \left\{Q_{\mathrm{B}}\,,\,\xi\left\{Q_{\mathrm{B}}\,,\,\tilde{\xi}c\tilde{c}e^{-\phi-\tilde{\phi}}V_3^{\prime\mathrm{DDF}}\right\}\right\} \times \prod_{r=4}^{N}\int d^2Z_r \prod_{r=4}^{N}\left\{Q_{\mathrm{B}}\,,\,\xi\left\{Q_{\mathrm{B}}\,,\,\tilde{\xi}e^{-\phi-\tilde{\phi}}V_3^{\prime\mathrm{DDF}}\right\}\right\} \times \prod_{r=1}^{N} e^{\frac{d-10}{16}\frac{i}{p_r^+}X^+}(z_I^{(r)},\bar{z}_I^{(r)})\Bigg\rangle$$

For sufficiently large $-d$, we do not encounter any divergences.

Dimensional regulzarization

In this form, the amplitude is not divergent even in the limit $d \longrightarrow 10$ and we obtain

$$\begin{aligned} \mathcal{A}_N \quad \sim \quad \Bigg\langle \prod_{r=1}^{2} \left[c\tilde{c} e^{-\phi-\tilde{\phi}} V_r'^{\mathrm{DDF}} \right] \\ \times \left\{ Q_{\mathrm{B}} \, , \, \xi \left\{ Q_{\mathrm{B}} \, , \, \tilde{\xi} c\tilde{c} e^{-\phi-\tilde{\phi}} V_3'^{\mathrm{DDF}} \right\} \right\} \\ \times \prod_{r=4}^{N} \int d^2 Z_r \ \left\{ Q_{\mathrm{B}} \, , \, \xi \left\{ Q_{\mathrm{B}} \, , \, \tilde{\xi} e^{-\phi-\tilde{\phi}} V_r'^{\mathrm{DDF}} \right\} \right\} \Bigg\rangle \end{aligned}$$

which coincides with the result of the first quantized formalism.
Therefore we get the right answer without adding any contact term interactions as counterterms.

§5 Outlook

- We have invented yet another way to realize string theories in noncritical dimensions.
- Dimensional regularization works without any contact term interactions.
- Ramond sector
- multi-loop amplitudes
- BRST invariant formulation $\alpha = 2p^{+}$ HKKO, covariantized light-cone,...