Light-cone gauge string field theory and dimensional regularization - Computation of FI D terms

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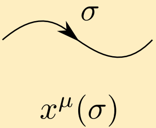
New frontiers in string theory 2018

String field theory (SFT)

String field



$$\longrightarrow \quad \Phi[x(\sigma)]$$

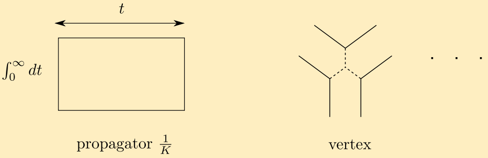
String field action

$$S = \Phi K \Phi + \Phi^3 + \cdots$$

- A nonperturbative definition of string theory
- We would like to discuss dynamical problems by SFT.

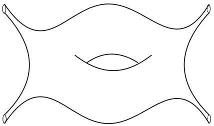
Amplitudes of superstring field theory

$$S = \Phi K \Phi + \Phi^3 + \cdots$$



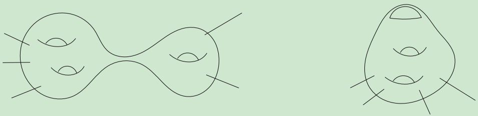
- Amplitudes can be calculated perturbatively.
- The results should coincide with the one from the first quantized theory.

$$A = \sum_{\text{worldsheet}}$$



Divergences of the Feynman amplitudes

1. Infrared divergences (physical)



2. Spurious singularities (unphysical)

- No ultraviolet divergences
- A valid superstring field theory should be free of the divergences of the second kind.

Light-cone gauge superstring field theory

Thise divergences can be regularized by formulating the theory in noncritical dimensions.

In this talk, I would like to explain

- how the regularization works
- computation of Fayet-Iliopoulos D terms using the formulation

Based on collaborations with Baba and Murakami and N. I. in progress

Outline

1. Divergences of Feynman amplitudes for superstrings
2. Light-cone gauge superstring field theory
3. Computation of Fayet-Iliopoulos D terms
4. Conclusions and discussions

§1 Divergences of Feynman amplitudes for superstrings

$$A = \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1 \cdots V_N \prod_\alpha \int_{C_\alpha} b \prod_i X(z_i) \right\rangle + \cdots \right](t)$$

M : moduli space of the Riemann surface

The integrand becomes singular at

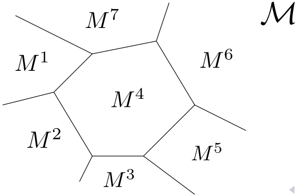
1. $t = t_0 \in \partial\mathcal{M}$: infrared divergences
2. $t = t_0 \notin \partial\mathcal{M}$: spurious singularities

- In the 1-st quantized formalism, this expression is derived by fixing the local symmetries on the worldsheet. GO
- The integrand diverges at the point where the gauge slice is not transverse to the gauge orbit. GO

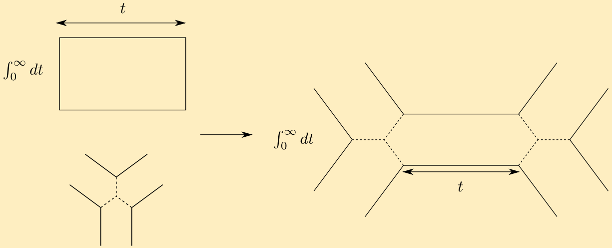
Spurious singularities in 1-st quantized formalism

$$A = \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1 \cdots V_N \prod_\alpha \int_{C_\alpha} b \prod_i X\left(z_i\left(t\right)\right) \right\rangle + \cdots \right](t)$$

- In practice, it is difficult to find a goood gauge slice everywhere in $\mathcal{M}$. One practical way to get such a slice is to divide $\mathcal{M}$ into patches. (Sen-Witten)
  - It is possible to find a good slice in each patch.
  - One can get and expression of A with contributions from the boundaries of the patches.



Spurious singularities in SFT



- SFT amplitudes coincide with those from the 1-st quantized approach.
- An SFT corresponds to a specific choice of the gauge slice.
- The Feynman rule of SFT should yield a good gauge slice for any Riemann surface.

Sen's SFT for superstrings

$$S = \frac{1}{g_s^2}\left[-\frac{1}{2}\left\langle\tilde{\Psi}\right| c_0^- Q_B \mathcal{G} \left|\tilde{\Psi}\right\rangle + \left\langle\tilde{\Psi}\right| c_0^- Q_B \left|\Psi\right\rangle + \sum_{n=1}^{\infty}\{\{\Psi^n\}\}\right]$$

- master action in BV formalism
- infinitely many interaction terms of order $\hbar^k \;\; (k = 0, 1, 2, \cdots)$
- One can arrange these interaction terms so that the amplitudes are free of spurious singularities order by order in $\hbar$.

§2 Light-cone gauge superstring field theory

LC gauge string field

$$\begin{cases} X^+ = t \\ \psi^+ = 0 \end{cases} \longrightarrow \Phi\left[t, \alpha, X^i(\sigma), \psi^i(\sigma), \lambda^A(\sigma)\right]$$

- Lorentz invariance, supersymmetry, etc. are not manifest
- Simple SFT action GO
- Tractable spurious singularities GO
  - We should deal with only the contact term divergences

Dimensional regularization

LC SFT can be formulated in any spacetime dimensions.

$$S = \int \left[\frac{1}{2} \Phi \cdot (i\alpha\partial_t - H)\,\Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$

$$A = \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\mathrm{LC}} \cdots V_N^{\mathrm{LC}} \prod_{I=1}^{2g-2+N} \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right] \right\rangle_{X^i,\psi^i} e^{-\frac{d-2}{8}\Gamma}$$

- $e^{-\Gamma}$ diverges when the LC diagram becomes singular. GO
- Taking d to be large and negative, divergences are regularized.
- $i\alpha\partial_t - H \sim p^2 - m^2 - \frac{10-d}{8}$: $\frac{10-d}{8}$ works as an infrared regulator
- Chiral fermions are dealt with by considering a linear dilaton background.

GO

$Q^2 \sim \frac{10-d}{8} \to 0$

$$
\begin{aligned}
A &= \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\mathrm{LC}} \cdots V_N^{\mathrm{LC}} \prod_{I=1}^{2g-2+N} \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}\left(z_I\right) \right] \right\rangle_{\mathrm{LC}} e^{-(1-Q^2)\Gamma} \\
&= \sum_{\text{spin structure}} \int_{\mathcal{M}} \prod_K dt_K \left\langle V_1 \cdots V_N \prod_{\alpha} \int_{C_K} b \prod_I X\left(z_I\right) \right\rangle
\end{aligned}
$$

- The amplitudes can be expressed using a conformal gauge worldsheet theory with $Q_B^2 = 0$. ▸ GO
- The expression of A is BRST invariant. $\to$ the regularization preserves the gauge invariance
- The conformal gauge expression coincides with the one from the 1-st quantized formalism in the limit $Q \to 0$, if the latter is (absolutely) convergent.

§3 Computation of Fayet-Iliopoulos D terms

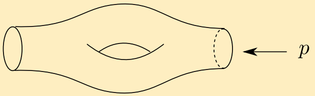
SO(32) hetertic string theory compactified on a CY-manifold with $A_i = \omega_i$

With anomalous $U(1)$'s, FI D terms appear at one loop

$$\begin{aligned} V &= -\frac{1}{2}D^2 + D\left(cg_\mathrm{s}^2 - |\phi|^2\right) + \cdots \\ &\to \frac{1}{2}\left(cg_\mathrm{s}^2 - |\phi|^2\right)^2 + \cdots \end{aligned}$$

- The supersymmetric vacuum is at $|\phi|^2 = cg_\mathrm{s}^2$
- $c > 0$ can be obtained by calculating the tachyonic mass $m^2 = -cg_\mathrm{s}^2$ of ϕ at the classical vacuum $\phi = 0$. (Dine-Seiberg-Witten, Dine-Ichinose-Seiberg, Atick-Dixon-Sen, Green-Seiberg, ..., Witten, Sen)

Computation of the m^2

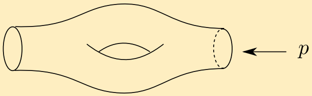


- One loop mass correction

$$
\begin{aligned}
\Sigma\left(p^2\right)\Big|_{p^2=0} &\sim \int d^2\tau d^2 z \left\langle V^{(0)}(z,\bar{z})\, V^{(0)}(0,0)\right\rangle\Big|_{p^2=0} \\
&\sim \int d^2\tau d^2 z \left[p^2 \,|z|^{-2-2p^2} \left\langle V_D(0,0)\right\rangle\right]\Big|_{p^2=0} \\
&\sim \int d^2\tau \left\langle V_D(0,0)\right\rangle
\end{aligned}
$$

- Sen's SFT reproduces this result.

Computation of the m^2 by LC SFT



- With the infrared regulator $Q^2 \sim \frac{10-d}{8} \to 0$

$$\begin{aligned} \Sigma\left(p^2\right)\Big|_{p^2=0} &\sim \int d^2\tau d^2z \left\langle V^{(0)}(z,\bar{z})\, V^{(0)}(0,0)\right\rangle\Big|_{p^2=0} \\ &\sim \int d^2\tau d^2z \left[|z|^{-3Q^2}\,\bar{z}^{-1}\left\langle\psi^-(z)\,\psi^-(0)\right\rangle\langle V_D(0,0)\rangle\right]\Big|_{p^2=0} \\ &\sim \int d^2\tau\,\langle V_D(0,0)\rangle\, f(\tau) \end{aligned}$$

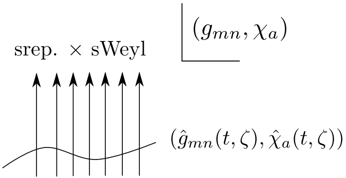
- We have not checked if this agrees with the known result.

§4 Conclusions and discussions

- In order to regularize the divergences of the Feynman amplitudes, we formulate light-cone gauge superstring field theory in noncritical dimensions.
- Taking $d \to 10$, we obtain the amplitudes which coincide with those from the first quantized approach.
- FI D terms can be calculated using the formalism.

1-st quantized amplitudes ▸ BACK

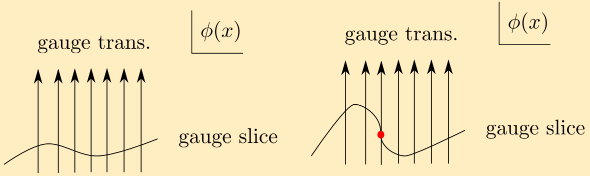
$$
\begin{aligned}
A &= \int \frac{[dg_{mn} d\chi_a dX^\mu d\psi^\mu]}{\text{superrep.} \times \text{superWeyl}} e^{-I} V_1 \cdots V_N \\
&= \int \prod_K dt_K \left[dX^\mu dbdcd\beta d\gamma\right] e^{-I_{\text{g.f.}}} \left[V_1 \cdots V_N \prod_K \int_{C_K} b \prod_i X(z_i) + \cdots\right] \\
&= \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1 \cdots V_N \prod_\alpha \int_{C_K} b \prod_i X(z_i) \right\rangle + \cdots\right]
\end{aligned}
$$



$$
\begin{aligned}
\epsilon^m &\longleftrightarrow b, c \ \text{(reparametrization)} \\
\epsilon^a &\longleftrightarrow \beta, \gamma \ \text{(supersymmetry)} \\
X(z) &= \delta(\beta) T_F + \cdots
\end{aligned}
$$

picture changing operator

Gauge slice



- When the gauge slice is not transverse to the gauge orbit at some point on the gauge slice, the relevant ghost have zero modes and
  - $\triangle_{\rm FP} = 0$ if the ghost is Grassmann odd
  - $\triangle_{\rm FP} = \infty$ if the ghost is Grassmann even
- The integrand of the Feynman amplitude diverges when the gauge slice is bad and γ has zero modes.

Singularities ▸ BACK

$$A = \int_{\mathcal{M}} \prod_{\alpha} dt_{\alpha} \left[\left\langle V_1 \cdots V_N \prod_{\alpha} \int_{C_\alpha} b \prod_i X(z_i) \right\rangle + \cdots \right] (t)$$

$$\left\langle \prod_i \delta(\beta)(z_i) \prod_r \delta(\gamma)(Z_r) \right\rangle$$

$$\propto \frac{1}{\vartheta[\alpha]\left(\sum z_i - \sum Z_r - 2\triangle\right)} \cdot \frac{\prod_{i,r} E(z_i, Z_r)}{\prod_{i>j} E(z_i, z_j) \prod_{r>s} E(Z_r, Z_s)} \cdot \frac{\prod_r \sigma(Z_r)^2}{\prod_i \sigma(z_i)^2}$$

- Two kinds of singularities
  1. $z_i = z_j$: contact term divergence
  2. $\vartheta[\alpha]\left(\sum z_i - \sum Z_r - 2\triangle\right) = 0$
- The second one is harder to deal with. (global condition)

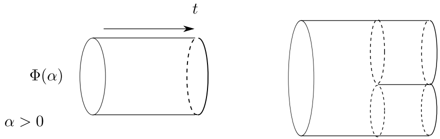
LC gauge SFT action ▸ BACK

$$S = \int \left[\frac{1}{2} \Phi \cdot (i\alpha\partial_t - H)\, \Phi + \frac{g_s}{3} \Phi \cdot (\Phi * \Phi) \right]$$

- String field $\Phi\left[t, \alpha, X^i(\sigma), \psi^i(\sigma)\right]$

$$\begin{aligned} t &= x^+ \\ \alpha &= 2p^+ \end{aligned}$$

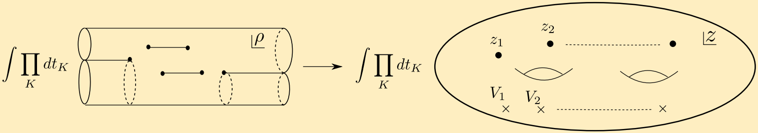
- propagator and vertex



Feynman amplitudes for LC gauge SFT

$$A = \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\mathrm{LC}} \cdots V_N^{\mathrm{LC}} \prod_I \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\mathrm{LC}}(z_I) \right] \right\rangle_{X^i, \psi^i} e^{-\Gamma}$$

$$= \sum_{\text{spin structure}} \int_{\mathcal{M}} \prod_K dt_K \left\langle V_1(Z_1) \cdots V_N(Z_N) \prod_K \int_{C_K} b \prod_I X(z_I) \right\rangle$$



- A naturally defined metric on LC diagram $ds^2 = d\rho d\bar{\rho}$
- $e^{-\Gamma}$: Weyl anomaly

Spurious singularities in LC SFT ▸ BACK

$$A = \sum_{\text{spin structure}} \int \prod_K dt_K \left\langle V_1^{\text{LC}} \cdots V_N^{\text{LC}} \prod_I \left[\left(\partial^2 \rho\right)^{-\frac{3}{4}} T_F^{\text{LC}}(z_I) \right] \right\rangle_{X^i, \psi^i} e^{-\Gamma}$$

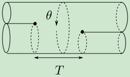
$$= \sum_{\text{spin structure}} \int_{\mathcal{M}} \prod_K dt_K \left[\left\langle V_1(Z_1) \cdots V_N(Z_N) \prod_K \int_{C_K} b \prod_I X(z_I) \right\rangle + \cdots \right]$$

1. $z_I = z_J$
2. $\vartheta[\alpha]\left(\sum z_I - \sum Z_r - 2\triangle\right) = 0$

- No singularity of the second type.
  - No β, γ on the worldsheet (1-st line)
  - The ϑ is canceled by the one from the $\psi^{\pm}$ partition function (2-nd line)

Singular LC diagrams ▸ BACK

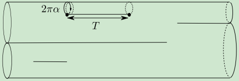
Contact term



$T \to 0$

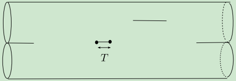
$\theta \to \theta_0$

Infinitely thin cylinder

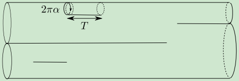


$\alpha \to 0$

Tiny neck



$T \to 0$



$T \sim \alpha \sim \epsilon \sim 0$

- $e^{-\Gamma}$ becomes singular when combinations of these phenomena happen.
- These correspond to contact term and infrared divergences.

Problems with chiral fermions ▸ BACK

- Naive dimensional regularization has problems with chiral fermions. We can avoid them by considering the theory in linear dilaton background $\Phi = -iQX^1$, instead of changing the spacetime dimensions

$$S = \frac{1}{16\pi}\int d^2z\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a X^1\partial_b X^1 - 2iQ\hat{R}X^1 + \cdots\right)$$

  - Doing so does not change the number of $\psi^\mu \sim \gamma^\mu$
  - $Q^2 \sim \frac{10-d}{8}$
  - We can change Q continuously.
  - This background breaks unitarity.

$X^{\pm}$ CFT

The worldsheet theory for $X^{\pm}, \psi^{\pm}$

$$S_{\pm} = -\frac{1}{2\pi}\int d^2 z \partial X^{+}\bar{\partial}X^{-} - \frac{d-10}{32\pi}\int d^2 z\left(\partial\chi\bar{\partial}\chi + \hat{g}_{z\bar{z}}\hat{R}\chi\right) + \cdots$$
$$\chi \equiv \ln\left(-4\partial X^{+}\bar{\partial}X^{+}\right) - \ln\left(2\hat{g}_{z\bar{z}}\right)$$

- This theory can be formulated in the case $\langle \partial_m X^{+} \rangle \neq 0$.
- In the case of the LC gauge amplitudes, we always have $\prod e^{-ip_r^{+}X^{-}} \quad (p_r^{+} \neq 0)$ and $\langle \partial_m X^{+} \rangle \neq 0$.

$X^{\pm}$ CFT

BACK

$$
\begin{aligned}
S_{\pm} &= -\frac{1}{2\pi}\int d^2 z \partial X^+ \bar{\partial} X^- - \frac{d-10}{32\pi}\int d^2 z \left(\partial\chi\bar{\partial}\chi + \hat{g}_{z\bar{z}}\hat{R}\chi\right) + \cdots \\
T(z) &= :\partial X^-(z)\partial X^+(z): - \frac{d-10}{8}\left[\frac{\partial^3 X^+}{\partial X^+} - \frac{3}{2}\left(\frac{\partial^2 X^+}{\partial X^+}\right)^2\right]
\end{aligned}
$$

- This theory is exactly solvable and turns out to be a superconformal field theory with $c = 3 + \frac{3}{2}(10-d)$.
- The worldsheet theory has a nilpotent BRST charge

$$
\begin{array}{ccccccccc}
 & & X^{\pm} & & X^i & & \text{ghosts} & & \\
c & = & 3 + \frac{3}{2}(10-d) & + & \frac{3}{2}(d-2) & - & 15 & = & 0
\end{array}
$$