Dimensional regularization of light-cone gauge superstring field theory and multiloop amplitudes

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String Field Theory

SFT should reproduce the scattering amplitudes calculated by the first-quantized formalism.

In this talk, I would like to explain that using light-cone gauge SFT for closed superstrings (NSR formalism)

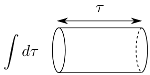
- it is possible to calculate multiloop amplitudes
- the results coincide with those of the first-quantized formalsim (Sen-Witten prescription arXIv:1504.00609)
  - NS-NS sector external lines, even spin structure (parity nonviolating amplitudes)

§1 Light-cone gauge super SFT

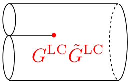
$$S = \int \left[\frac{1}{2}\Phi \cdot \left(i\partial_t - \frac{L_0 + \tilde{L}_0 - 1}{p^+} \right) \Phi + \frac{g}{3}\Phi \cdot (\Phi * \Phi) \right] + (\text{Ramond sectors})$$

Feynmn rule

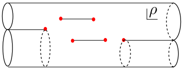
- propagator

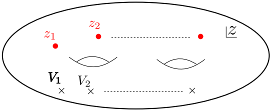


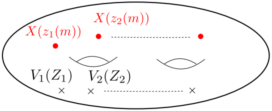
- vertex BACKUP



Amplitudes BACKUP

$\int \prod_{\mathcal{I}} dt_{\mathcal{I}}$ 

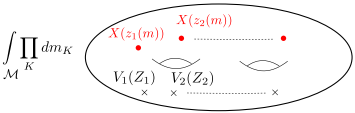
$\longrightarrow \int \prod_{\mathcal{I}} dt_{\mathcal{I}}$ 

$= \int_{\mathcal{M}} \prod_{K} dm_K$ 

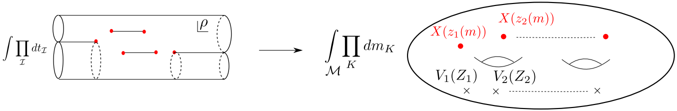
$$
\begin{aligned}
A^{\mathrm{LC}} &= \sum_{\text{Channels}} \int \prod_{\mathcal{I}} dt_{\mathcal{I}} \left\langle \prod_{I=1}^{2g-2+N} \left| (\partial^2 \rho)^{-\frac{3}{4}} G^{\mathrm{LC}}(z_I) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle^{X^i, \psi^i} e^{-\Gamma} \\
&= \int_{\mathcal{M}} \prod_{K} dm_K \left\langle \prod_{K} \oint (\mu_K b + \bar{\mu}_K \bar{b}) \prod_{I=1}^{2g-2+N} X(z_I) \bar{X}(\bar{z}_I) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle^{X^\mu, \psi^\mu, \text{ghosts}}
\end{aligned}
$$

§2 First-quantized formalism

The amplitudes can be expressed by using the picture changing operator X.



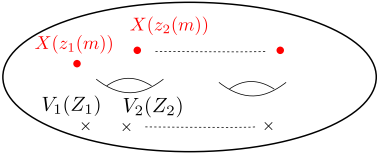
- (Naively) the amplitude does not depend on the way to put PCO's.



- The light-cone gauge amplitude corresponds to a specific one.

Spurious singularities

$$\int_{\mathcal{M}} \prod_K dm_K$$



- The integrand diverges when $z_i(m)$'s satisfy some conditions. BACKUP
- There is no (known) global way to choose $z_i(m)$ to avoid these singularities.

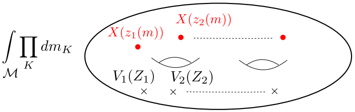
Because of these singularities, this expression is not well-defined.

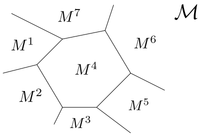
(Verlinde-Verlinde)

Sen-Witten prescription

One can define amplitudes using PCO's avoiding singularities. (Sen, Sen-Witten)

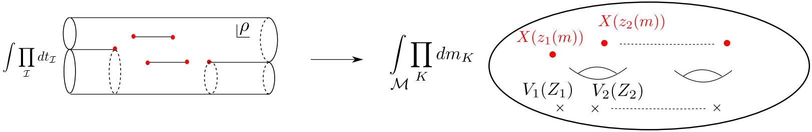
- Divide the moduli space $\mathcal{M}$ into polyhedra M^α.





- In each M^α, one can put the PCO's avoiding spurious singularities.
- The amplitude is given as a sum of contributions from $\mathcal{M}^\alpha, \partial\mathcal{M}^\alpha, \ldots$.
- The result does not depend on the choices made.

§3 Dimensional regularization

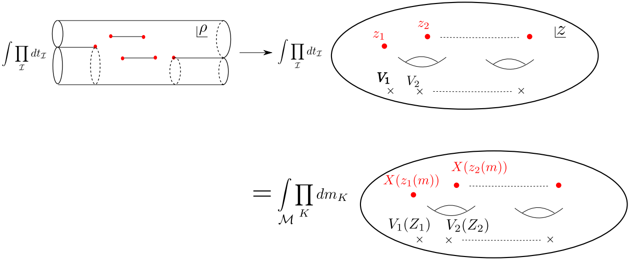


- The choice of $z_i(m)$ is fixed in the LCSFT.
- One can regularize the singularities by dimensional regularization.
  - A_d^{LC} can be defined as an analytic function of d for $-d$ large enough.
  - $\lim_{d \to 10} A_d^{\mathrm{LC}}$ can be shown to coincide with the first-quantized result.

Dimensional regularization

Light-cone gauge SFT can be formulated in any d. BACKUP

$$S=\int\left[\frac{1}{2}\Phi\cdot\left(i\partial_t-\frac{L_0+\tilde{L}_0-\frac{d-2}{8}}{p^+}\right)\Phi+\frac{g}{3}\Phi\cdot(\Phi*\Phi)\right]+(\text{Ramond sectors})$$



Amplitudes

$$
\begin{aligned}
A_d^{\mathrm{LC}} &= \int \prod_{\mathcal{I}} dt_{\mathcal{I}} \left\langle \prod_{I=1}^{2g-2+N} \left| \left(\partial^2 \rho\right)^{-\frac{3}{4}} G^{\mathrm{LC}}\left(z_I\right) \right|^2 \prod_{r=1}^{N} V_r^{\mathrm{LC}} \right\rangle e^{-\frac{d-2}{16}\Gamma} \\
&= \int_{\mathcal{M}} \prod_K dm_K \left\langle \prod_K \oint \left(\mu_K b + \bar{\mu}_K \bar{b}\right) \prod_{I=1}^{2g-2+N} X(z_I)\bar{X}(\bar{z}_I) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle^{\mathrm{conf.}}
\end{aligned}
$$

- The spurious singularities and the infrared divergences of the superstring theory can be regularized by choosing the worldsheet theory appropriately and taking $-d$ large enough.

- BRST invariant worldsheet theory with a nontrivial CFT for $X^{\pm}, \psi^{\pm}$ ($X^{\pm}$ CFT). BACKUP

$d \to 10$

$$A_d^{\mathrm{LC}} = \int_{\mathcal{M}} \prod_K dm_K \left\langle \prod_K \oint \left(\mu_K b + \bar{\mu}_K \bar{b}\right) \prod_{I=1}^{2g-2+N} X(z_I)\bar{X}(\bar{z}_I) \prod_{r=1}^{N} V_r^{\mathrm{conf.}} \right\rangle^{\mathrm{conf.}}$$

- One can also define the amplitude A_d^{SW} following the Sen-Witten prescription using the BRST invariant worldsheet theory for $d \neq 10$.
- $A_d^{\mathrm{SW}} = A_d^{\mathrm{LC}}$ for $-d$ large enough because
  - A_d^{SW} does not depend on the way to put PCO's as long as it avoids singularities
  - there are no singularities for $-d$ large enough

Therefore $\lim_{d\to 10} A_d^{\mathrm{LC}}$ coincides with the first-quantized amplitude A_{10}^{SW}.

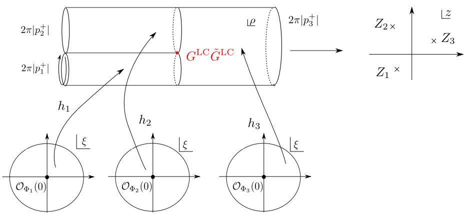
§4 Conclusions and discussions

- Dimensional regularization of the light-cone gauge super SFT can be used to reproduce the results of the first-quantized formalism.
- The dimensional regularization also works as the regularization of the infrared divergences.

- Ramond sector, odd spin structure
- Dimensional regularization in Witten's cubic SFT?
- Nonperturbative calculations by SFT?

Backup

Three-string vertex ▸ BACK



The total central charge is 12 and we have to include the anomaly factor

$$\int \Phi_1 \cdot (\Phi_2 * \Phi_3) = \int dt \prod_{r=1}^{3} \left(\frac{p_r^+ dp_r^+}{4\pi} \right) \delta \left(\sum_{r=1}^{3} p_r^+ \right) \left(p_1^+ p_2^+ p_3^+ \right)^{-\frac{1}{2}} e^{-\sum_r \frac{1}{p_r^+} \sum_{s=1}^{3} p_s^+ \ln |p_s^+|}$$
$$\times \left\langle \left| \partial^2 \rho\left(z_I\right) \right|^{-\frac{3}{2}} G^{\mathrm{LC}}\left(z_I\right) \bar{G}^{\mathrm{LC}}\left(\bar{z}_I\right)$$
$$\times\ \rho^{-1} h_1 \circ \mathcal{O}_{\Phi_1(t,\alpha_1)} \rho^{-1} h_2 \circ \mathcal{O}_{\Phi_2(t,\alpha_2)} \rho^{-1} h_3 \circ \mathcal{O}_{\Phi_3(t,\alpha_3)} \right\rangle_{\mathbb{C}}$$

PCO

$$
\begin{aligned}
X(z) &= \{Q_{\mathrm{B}}, \xi(z)\} \\
&= -e^{\phi}G + c\partial\xi + \frac{1}{4}\partial b\eta e^{2\phi} + \frac{1}{4}\left(2\partial\eta e^{2\phi} + \eta\partial e^{2\phi}\right) \\
\beta(z) &= e^{-\phi}\partial\xi(z) \\
\gamma(z) &= \eta e^{\phi}(z) \\
\delta(\beta) &= e^{\phi} \\
\delta(\gamma) &= e^{-\phi}
\end{aligned}
$$

BACK

Spurious singularities ▸ BACK

Superghost correlation function

$$\left\langle \prod_i e^{\phi}(z_i) \prod_r e^{-\phi}(Z_r) \right\rangle$$

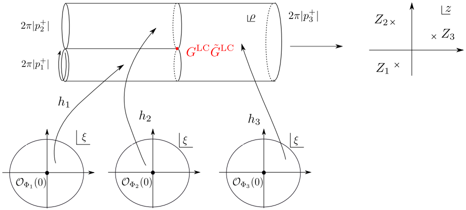
$$\propto \frac{1}{\vartheta[\alpha]\left(\sum z_i - \sum Z_r - 2\triangle\right)} \cdot \frac{\prod_{i,r} E(z_i, Z_r)}{\prod_{i>j} E(z_i, z_j) \prod_{r>s} E(Z_r, Z_s)} \cdot \frac{\prod_r \sigma(Z_r)^2}{\prod_i \sigma(z_i)^2}$$

diverges when

$$\begin{cases} z_i = z_j \\ \vartheta[\alpha]\left(\sum z_i - \sum Z_r - 2\triangle\right) = 0 \end{cases}$$

At these points, the gauge fixing is not good.

Three-string vertex ▸ BACK



The total central charge is $\frac{3}{2}(d-2)$ and the anomaly factor becomes

$$
\begin{aligned}
\int \Phi_1 \cdot (\Phi_2 * \Phi_3) \quad = \quad & \int dt \prod_{r=1}^{3} \left(\frac{p_r^+ dp_r^+}{4\pi} \right) \delta \left(\sum_{r=1}^{3} p_r^+ \right) \\
& \times \left(p_1^+ p_2^+ p_3^+ \right)^{-\frac{d-2}{16}} e^{-\frac{d-2}{8} \sum_r \frac{1}{p_r^+} \sum_{s=1}^3 p_s^+ \ln \left| p_s^+ \right|} \\
& \times \left\langle \left| \partial^2 \rho (z_I) \right|^{-\frac{3}{2}} G^{\mathrm{LC}}(z_I) \bar{G}^{\mathrm{LC}}(\bar{z}_I) \right. \\
& \quad \times \left. \rho^{-1} h_1 \circ \mathcal{O}_{\Phi_1(t,\alpha_1)} \rho^{-1} h_2 \circ \mathcal{O}_{\Phi_2(t,\alpha_2)} \rho^{-1} h_3 \circ \mathcal{O}_{\Phi_3(t,\alpha_3)} \right\rangle_{\mathbb{C}}
\end{aligned}
$$

$X^{\pm}$ CFT

$$S_{X^\pm} = -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}X^+DX^- + \bar{D}X^-DX^+\right) + \frac{d-10}{8}\Gamma_{\mathrm{super}}[\Phi]$$

$$X^\pm \equiv x^\pm + i\theta\psi^\pm + i\bar{\theta}\tilde{\psi}^\pm + i\theta\bar{\theta}F^\pm$$

$$\Gamma_{\mathrm{super}}[\Phi] = -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}\Phi D\Phi + \theta\bar{\theta}\hat{g}_{z\bar{z}}\hat{R}\Phi\right)$$

$$\Phi \equiv \ln\left|\partial X^+ - \frac{\partial DX^+DX^+}{(\partial X^+)^2}\right|^2 - \ln\hat{g}_{z\bar{z}}$$

- This theory can be formulated in the case $\langle\partial_m X^+\rangle \neq 0$.
- In the case of the LC gauge amplitudes, we always have $\prod e^{-ip_r^+X^-}$ $(p_r^+ \neq 0)$ and $\langle\partial_m X^+\rangle \neq 0$.

$X^{\pm}$ CFT

$$
\begin{aligned}
S_{X^{\pm}} &= -\frac{1}{2\pi}\int d^2z d\theta d\bar{\theta}\left(\bar{D}X^{+}DX^{-}+\bar{D}X^{-}DX^{+}\right)+\frac{d-10}{8}\Gamma_{\mathrm{super}}\left[\Phi\right] \\
T\left(z,\theta\right) &= G\left(z\right)+\theta T\left(z\right) \\
&= \frac{1}{2}:\partial X^{+}DX^{-}\left(\mathbf{z}\right):+\frac{1}{2}:DX^{+}\partial X^{-}\left(\mathbf{z}\right):-\frac{d-10}{4}S\left(\mathbf{z},\boldsymbol{X}^{+}\right)
\end{aligned}
$$

- It is a superconformal field theory with $\hat{c}=12-d$.
- The worldsheet theory becomes BRST invariant

		$X^{\pm}$		X^{i}		ghosts		
$\hat{c}$	$=$	$12-d$	$+$	$d-2$	$-$	10	$=$	0

$X^{\pm}$ CFT

$$
\begin{aligned}
T(z,\theta) &= G(z) + \theta T(z) \\
&= \frac{1}{2} : \partial X^{+} D X^{-}(\mathbf{z}) : + \frac{1}{2} : D X^{+} \partial X^{-}(\mathbf{z}) : - \frac{d-10}{4} S\left(\mathbf{z}, \boldsymbol{X}^{+}\right)
\end{aligned}
$$

$$
\begin{aligned}
S\left(\mathbf{z}, \boldsymbol{X}^{+}\right) &= \frac{\partial^{2}\Theta^{+}}{D\Theta^{+}} - \frac{2\partial D\Theta^{+}\partial\Theta^{+}}{\left(D\Theta^{+}\right)^{2}} \\
\Theta^{+} &= \frac{D X^{+}}{\left(\partial X^{+}\right)^{\frac{1}{2}}} \\
\mathbf{z} &= (z,\theta) \\
D &= \frac{\partial}{\partial\theta} + \theta\frac{\partial}{\partial z}
\end{aligned}
$$

$X^{\pm}$ CFT

One can calculate the OPE's:

$$
\begin{aligned}
X^{+}(\mathbf{z})\, X^{+}(\mathbf{z}') &\sim \text{regular} \\
X^{+}(\mathbf{z})\, X^{-}(\mathbf{z}') &\sim \ln\left|\mathbf{z}-\mathbf{z}'\right|^{2} \\
X^{-}(\mathbf{z})\, X^{-}(\mathbf{z}') &\sim -\frac{d-10}{4}\left[\frac{\theta-\theta'}{(\mathbf{z}-\mathbf{z}')^{3}}\frac{3DX^{+}}{\left(\partial X^{+}\right)^{3}}(\mathbf{z}')\right. \\
&\quad+\frac{1}{(\mathbf{z}-\mathbf{z}')^{2}}\left(\frac{1}{2\left(\partial X^{+}\right)^{2}}+\frac{4\partial DX^{+}DX^{+}}{\left(\partial X^{+}\right)^{4}}\right)(\mathbf{z}') \\
&\quad+\frac{\theta-\theta'}{(\mathbf{z}-\mathbf{z}')^{2}}\left(-\frac{\partial DX^{+}}{\left(\partial X^{+}\right)^{3}}-\frac{5\partial^{2}X^{+}DX^{+}}{2\left(\partial X^{+}\right)^{4}}\right)(\mathbf{z}') \\
&\quad+\frac{1}{\mathbf{z}-\mathbf{z}'}\left(-\frac{\partial^{2}X^{+}}{2\left(\partial X^{+}\right)^{3}}+\frac{2\partial^{2}DX^{+}DX^{+}}{\left(\partial X^{+}\right)^{4}}-\frac{8\partial^{2}X^{+}\partial DX^{+}DX^{+}}{\left(\partial X^{+}\right)^{5}}\right)(\mathbf{z}') \\
&\quad+\frac{\theta-\theta'}{\mathbf{z}-\mathbf{z}'}\left(-\frac{\partial^{2}DX^{+}}{2\left(\partial X^{+}\right)^{3}}+\frac{3\partial^{2}X^{+}\partial DX^{+}}{2\left(\partial X^{+}\right)^{4}}-\frac{\partial^{3}X^{+}DX^{+}}{2\left(\partial X^{+}\right)^{4}}\right. \\
&\qquad\left.\left.+\frac{\left(\partial^{2}X^{+}\right)^{2}DX^{+}}{\left(\partial X^{+}\right)^{5}}-\frac{\partial^{2}DX^{+}\partial DX^{+}DX^{+}}{\left(\partial X^{+}\right)^{5}}\right)(\mathbf{z}')\right]
\end{aligned}
$$

$X^{\pm}$ CFT ▸ BACK

From these, it is possible to prove

$$T(\mathbf{z})\,T(\mathbf{z}') \sim \frac{12-d}{4(\mathbf{z}-\mathbf{z}')^3} + \frac{\theta-\theta'}{(\mathbf{z}-\mathbf{z}')^2}\frac{3}{2}T(\mathbf{z}') + \frac{1}{\mathbf{z}-\mathbf{z}'}\frac{1}{2}DT(\mathbf{z}') + \frac{\theta-\theta'}{\mathbf{z}-\mathbf{z}'}\partial T(\mathbf{z}')$$

$T(\mathbf{z})$ satisfies the super Virasoro algebra with $\hat{c} = 12 - d$.